

On New Intuitionistic Fuzzy Generalized Closed Mappings In Intuitionistic Fuzzy Topological Spaces

S. Pious Missier¹, S. Gabriel Raja^{2*}, and J. Martina Jency³

¹Head & Associate Professor (Retd), Department of Mathematics,
Don Bosco College of Arts and Science,

(Affiliated to Mononmaniam Sundaranar University, Abishekapatti, Tirunelveli – 627 012)
Keela Eral, Thoothukudi – 628 908, Tamil Nadu, India. smissier@gmail.com

^{2*}Research Scholar (Reg. No – 19222052091008), G. Venkataswamy Naidu College,
(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012),
Kovilpatti, Tamil Nadu – 628 502, India. gabrirlraja@gmail.com

³Assistant Professor, Department of Mathematics, V. O. Chidambaram College
(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli- 627 012),
Thoothukkudi, Tamil Nadu – 628 008, India. martina.mat@voccollege.ac.in

This paper studies the concepts of intuitionistic fuzzy generalized hash pre (briefly, $I_{fg_p}^\#$) closed mappings in intuitionistic fuzzy topological spaces. It also investigates some of its characteristics and properties.

Key Words: Intuitionistic fuzzy mappings, Intuitionistic fuzzy $g_p^\#$ closed mappings

1. INTRODUCTION

L.A. Zadeh [26] introduced fuzzy sets to value every member of a set in mathematics in 1965. C.L. Chang [7] introduced fuzzy topological spaces in 1968. Further, K. Atanassov [3] introduced intuitionistic fuzzy sets by adding the notion of non-membership into fuzzy sets in 1983. D. Coker [9] introduced intuitionistic fuzzy topological spaces in 1997. From then on, many research works focused on the generalization of the concepts of intuitionistic fuzzy topological spaces. Extending further, Pious Missier .S and Gabriel Raja .S [17] introduced intuitionistic fuzzy generalized hash pre-closed (briefly, $I_{fg_p}^\#C$) set with its characterization and properties with suitable examples. The aim of this paper is to study and investigate $I_{fg_p}^\#C$ mappings with suitable examples and theorem.

2. PRELIMINARIES

Definition 2.1 [3] Let X be a universal set and let A be an intuitionistic fuzzy (briefly, I_f) subset in X , where $A = \{ \langle x/\mu_A(x), \nu_A(x) \rangle : x \in X \}$. Here, the functions $\mu_A: X \rightarrow [0,1]$ and $\nu_A: X \rightarrow [0,1]$ denote the degree of membership, namely $\mu_A(x)$ and the degree of non-membership, namely $\nu_A(x)$ of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2 [9] An I_f topology on a non-empty set X is a family τ_{if} of intuitionistic fuzzy subsets of X , satisfying the following axioms;

- (1) $\tilde{0}, \tilde{1} \in \tau_{if}$
- (2) $A \cap B \in \tau_{if}$ for any $A, B \in \tau_{if}$
- (3) $\cup A_i \in \tau_{if}$ for any arbitrary family $\{A_i: i \in J\} \subseteq \tau_{if}$

A non-empty set X on which an I_f topology τ_{if} has been specified is called an intuitionistic fuzzy topological space, i.e., (X, τ_{if}) . Any I_f set in τ_{if} is known as an intuitionistic fuzzy open (briefly, I_f open) set in X and the complement of an I_fO set is known as an intuitionistic fuzzy closed (briefly, I_f closed) set in X .

Definition 2.3 [9] Let (X, τ_{if}) be an I_f topological space and $A = \{ \langle x/\mu_A(x), \nu_A(x) \rangle : x \in X \}$ be an I_f subset in X . Then the interior and closure of the above I_f subset are defined as follows.

- (i) $\text{int}(A) = \cup \{G | G \text{ is an } I_fO \text{ set in } X \text{ and } G \subseteq A\}$
- (ii) $\text{cl}(A) = \cap \{K | K \text{ is an } I_fC \text{ set in } X \text{ and } A \subseteq K\}$

Definition 2.4 [12] Let (X, τ_{if}) be an intuitionistic fuzzy topological space (briefly, I_fTS). Then an intuitionistic fuzzy subset $L \subseteq X$ is said to be an

- (i) intuitionistic fuzzy semi-closed (briefly, I_fSC) set if $I_f\text{int}(I_f\text{cl}(L)) \subseteq L$.
- (ii) intuitionistic fuzzy pre-closed (briefly, I_fPC) set if $I_f\text{cl}(I_f\text{int}(L)) \subseteq L$.
- (iii) intuitionistic fuzzy alpha-closed (briefly, $I_f\alpha C$) set if $I_f\text{cl}(I_f\text{int}(I_f\text{cl}(L))) \subseteq L$.
- (iv) intuitionistic fuzzy semi-pre-closed (briefly, I_fspC) set [51] if $I_f\text{int}(I_f\text{cl}(I_f\text{int}(L))) \subseteq L$.

Definition 2.5 Let (X, τ_{if}) be an I_f topological space. Then an intuitionistic fuzzy subset $L \subseteq X$ is said to be an

- (i) intuitionistic fuzzy generalized closed (briefly, I_fgC) set [22] if $I_f\text{cl}(L) \subseteq U$ whenever $L \subseteq U$ and U is an I_f open set in (X, τ_{if}) .
- (ii) intuitionistic fuzzy semi generalized closed (briefly, I_fsgC) set [20] if $I_f\text{scl}(L) \subseteq U$ whenever $L \subseteq U$ and U is an I_fS open set in (X, τ_{if}) .
- (iii) intuitionistic fuzzy generalized pre-closed (briefly, I_fgpC) set [18] if $I_f\text{pcl}(L) \subseteq U$ whenever $L \subseteq U$ and U is an I_f open set in (X, τ_{if}) .
- (iv) intuitionistic fuzzy semi generalized pre-closed (briefly, I_fsgpC) set [6] if $I_f\text{pcl}(L) \subseteq U$ whenever $L \subseteq U$ and U is an I_fS open set in (X, τ_{if}) .
- (v) intuitionistic fuzzy generalized semi-pre-closed (briefly, I_fgspC) set [21] if $I_f\text{spcl}(L) \subseteq U$ whenever $L \subseteq U$ and U is an I_f open set in (X, τ_{if}) .

- (vi) intuitionistic fuzzy alpha generalized closed (briefly, $I_f\alpha gC$) set [19] if $I_f\alpha cl(L) \subseteq U$ whenever $L \subseteq U$ and U is an I_f open set in (X, τ_{if}) .
- (vii) intuitionistic fuzzy generalized alpha-closed (briefly, $I_fg\alpha C$) set [13] if $I_f\alpha cl(L) \subseteq U$ whenever $L \subseteq U$ and U is an $I_f\alpha$ open set in (X, τ_{if}) .
- (viii) intuitionistic fuzzy generalized star closed (briefly, I_fg^*C) set [4] if $I_fcl(L) \subseteq U$ whenever $L \subseteq U$ and U is an I_fg open set in (X, τ_{if}) .
- (ix) intuitionistic fuzzy generalized star pre-closed (briefly, I_fg^*pC) set [5] if $I_fpcl(L) \subseteq U$ whenever $L \subseteq U$ and U is an I_fg open set in (X, τ_{if}) .
- (x) intuitionistic fuzzy generalized star semi-closed (briefly, I_fg^*sC) set [16] if $I_fscL(L) \subseteq U$ whenever $L \subseteq U$ and U is an I_fg open set in (X, τ_{if}) .
- (xi) intuitionistic fuzzy omega closed (briefly, $I_f\omega C$) set [23] if $I_fcl(L) \subseteq U$ whenever $L \subseteq U$ and U is an I_fs open set in (X, τ_{if}) .
- (xii) intuitionistic fuzzy psi closed (briefly, $I_f\psi C$) set [15] if $I_fscL(L) \subseteq U$ whenever $L \subseteq U$ and U is an I_fsg open set in (X, τ_{if}) .
- (xiii) intuitionistic fuzzy generalized hash closed (briefly, $I_fg^\#C$) set [1] if $I_fcl(L) \subseteq U$ whenever $L \subseteq U$ and U is an $I_fg\alpha$ open set in (X, τ_{if}) .
- (xiv) intuitionistic fuzzy generalized hash pre closed (briefly, $I_fg_p^\#C$) set [17] if $I_fpcl(L) \subseteq U$ whenever $L \subseteq U$ and U is an $I_fg\alpha$ open set in (X, τ_{if}) .

The complement of I_fgC (I_fsgC , I_fgpC , I_fsgpC , I_fgspC , $I_f\alpha gC$, $I_fg\alpha C$, I_fg^*C , I_fg^*pC , I_fg^*sC , $I_f\omega C$, $I_f\psi C$, $I_fg^\#C$) set is called I_fgO (I_fsgO , I_fgpO , I_fsgpO , I_fgspO , $I_f\alpha gO$, $I_fg\alpha O$, I_fg^*O , I_fg^*pO , I_fg^*sO , $I_f\omega O$, $I_f\psi O$, $I_fg^\#O$, $I_fg_p^\#O$) set.

Definition 2.6 [9] Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an I_f function where $A = \{ \langle x / \mu_A(x), v_A(x) \rangle : x \in X \}$ and $B = \{ \langle y / \mu_B(y), v_B(y) \rangle : y \in Y \}$, then

- (i) the pre-image of B under f is denoted by $f^{-1}(B)$ and is defined by $f^{-1}(B) = \{ \langle x / f^{-1}(\mu_B)(x), f^{-1}(v_B)(x) \rangle : x \in X \}$ where $f^{-1}(\mu_B)(x) = \mu_B(f(x))$ and $f^{-1}(v_B)(x) = v_B(f(x))$.
- (ii) the image of A under f is denoted by $f(A)$ and is defined by $f(A) = \{ \langle y / f(\mu_B)(y), f_{-}(v_B)(y) \rangle : y \in Y \}$ where $f_{-}(v_B) = 1 - f(1 - v_B)$,

$$f(\mu_B)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$
and

$$f_{-}(v_B)(y) = \begin{cases} \inf_{x \in f^{-1}(y)} v_A(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}.$$

Definition 2.7 [14] Let (X, τ_{if}) and (Y, σ_{if}) be any two I_f topological spaces. Then an I_f mapping $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ is said to be an

- (i) intuitionistic fuzzy closed mapping, if the image of every I_f closed set in (X, τ_{if}) is an I_f closed set in (Y, σ_{if}) .

- (ii) intuitionistic fuzzy semi-closed mapping, if the image of every I_f closed set in (X, τ_{if}) is an I_f semi-closed set in (Y, σ_{if}) .
- (iii) intuitionistic fuzzy pre-closed mapping, if the image of every I_f closed set in (X, τ_{if}) is an I_f pre-closed set in (Y, σ_{if}) .
- (iv) intuitionistic fuzzy α -closed mapping, if the image of every I_f closed set in (X, τ_{if}) is an I_f α -closed set in (Y, σ_{if}) .

3. Intuitionistic Fuzzy $g_p^\#$ Closed Mappings

Definition 3.1 Let (X, τ_{if}) and (Y, σ_{if}) be any two I_f topological spaces. Then a mapping $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ is said to be an intuitionistic fuzzy generalized hash pre-closed (briefly, $I_f g_p^\#$ closed) mapping, if the image of every I_f closed set in (X, τ_{if}) is an $I_f g_p^\#$ closed set in (Y, σ_{if}) .

Example 3.2 Let (X, τ_{if}) and (Y, σ_{if}) be any two I_f topological spaces where $X = \{q, r\}$, $Y = \{s, t\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, $\sigma_{if} = \{\tilde{0}, B, \tilde{1}\}$, $A = \{< q/0.8, 0.2 >, < r/0.7, 0.3 >\}$ and $B = \{< s/0.3, 0.7 >, < t/0.4, 0.6 >\}$. Let's define a mapping $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ such that $f(q) = s$ and $f(r) = t$. Then an I_f set $L = \{< q/0.2, 0.8 >, < r/0.3, 0.7 >\}$ is an I_f closed set in (X, τ_{if}) . Clearly, we observe that $f(L)$ is an $I_f g_p^\#$ closed set in (Y, σ_{if}) . Hence f is an $I_f g_p^\#$ closed mapping.

Theorem 3.3 Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an I_f closed mapping. Then every I_f closed mapping is an $I_f g_p^\#$ closed mapping.

Proof: Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an I_f closed mapping and let L be an I_f closed set in (X, τ_{if}) . Since f is an I_f closed mapping, $f(L)$ is an I_f closed set in (Y, σ_{if}) . Moreover, every I_f closed set is an $I_f g_p^\#$ closed set. Therefore $f(L)$ is an $I_f g_p^\#$ closed set in (Y, σ_{if}) . Hence f is an $I_f g_p^\#$ closed mapping.

However, the converse of the above theorem is not always the case. This could be verified by the following example.

Example 3.4 Let (X, τ_{if}) and (Y, σ_{if}) be any two I_f topological spaces where $X = \{q, r\}$, $Y = \{s, t\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, $\sigma_{if} = \{\tilde{0}, B, \tilde{1}\}$, $A = \{< q/0.8, 0.2 >, < r/0.7, 0.3 >\}$ and $B = \{< s/0.3, 0.7 >, < t/0.4, 0.6 >\}$. Let's define an I_f mapping $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ such that $f(q) = s$ and $f(r) = t$. Then an I_f set $L = \{< q/0.2, 0.8 >, < r/0.3, 0.7 >\}$ is an I_f closed set in (X, τ_{if}) , but $f(L)$ is an $I_f g_p^\#$ closed set but not an I_f closed set in (Y, σ_{if}) . Hence, f is an $I_f g_p^\#$ closed mapping but not an I_f closed mapping.

Theorem 3.5 Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an $I_f\alpha$ closed mapping. Then every $I_f\alpha$ closed mapping is an $I_fg_p^\#$ closed mapping.

Proof: Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an $I_f\alpha$ closed mapping and let L be an I_f closed set in (X, τ_{if}) . Since f is an $I_f\alpha$ closed mapping, $f(L)$ is an $I_f\alpha$ closed set in (Y, σ_{if}) . Moreover, every $I_f\alpha$ closed set is an $I_fg_p^\#$ closed set. Therefore $f(L)$ is an $I_fg_p^\#$ closed set in (Y, σ_{if}) . Hence f is an $I_fg_p^\#$ closed mapping.

However, the converse of the above theorem is not always the case. This could be verified by the following example.

Example 3.6 Let (X, τ_{if}) and (Y, σ_{if}) be any two I_f topological spaces where $X = \{q, r\}$, $Y = \{s, t\}$, $\tau_{if} = \{\emptyset, A, \tilde{1}\}$, $\sigma_{if} = \{\emptyset, B, \tilde{1}\}$, $A = \{< q/0.6, 0.4 >, < r/0.5, 0.5 >\}$ and $B = \{< s/0.7, 0.3 >, < t/0.6, 0.4 >\}$. Let's define an I_f mapping $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ such that $f(q) = s$ and $f(r) = t$. Then an I_f set $L = \{< q/0.4, 0.6 >, < r/0.5, 0.5 >\}$ is an I_f closed set in (X, τ_{if}) , but $f(L)$ is an $I_fg_p^\#$ closed set but not an $I_f\alpha$ closed set in (Y, σ_{if}) . Hence, f is an $I_fg_p^\#$ closed mapping but not an $I_f\alpha$ closed mapping.

Theorem 3.7 Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an I_fp closed mapping. Then every I_fp closed mapping is an $I_fg_p^\#$ closed mapping.

Proof: Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an I_fp closed mapping and let L be an I_f closed set in (X, τ_{if}) . Since f is an I_fp closed mapping, $f(L)$ is an I_fp closed set in (Y, σ_{if}) . Moreover, every I_fp closed set is an $I_fg_p^\#$ closed set. Therefore $f(L)$ is an $I_fg_p^\#$ closed set in (Y, σ_{if}) . Hence f is an $I_fg_p^\#$ closed mapping.

However, the converse of the above theorem is not always the case. This could be verified by the following example.

Example 3.8 Let (X, τ_{if}) and (Y, σ_{if}) be any two I_f topological spaces where $X = \{q, r\}$, $Y = \{s, t\}$, $\tau_{if} = \{\emptyset, A, \tilde{1}\}$, $\sigma_{if} = \{\emptyset, B, \tilde{1}\}$, $A = \{< q/0.2, 0.8 >, < r/0.3, 0.7 >\}$ and $B = \{< s/0.7, 0.3 >, < t/0.6, 0.4 >\}$. Let's define an I_f mapping $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ such that $f(q) = s$ and $f(r) = t$. Then an I_f set $L = \{< q/0.8, 0.2 >, < r/0.7, 0.3 >\}$ is an I_f closed set in (X, τ_{if}) , but $f(L)$ is an $I_fg_p^\#$ closed set but not an I_fp closed set in (Y, σ_{if}) . Hence, f is an $I_fg_p^\#$ closed mapping but not an I_fp closed mapping.

Theorem 3.9 Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an I_fg^* closed mapping. Then every I_fg^* closed mapping is an $I_fg_p^\#$ closed mapping.

Proof: Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an I_fg^* closed mapping and let L be an I_f closed set in (X, τ_{if}) . Since f is an I_fg^* closed mapping, $f(L)$ is an I_fg^* closed set in (Y, σ_{if}) . Moreover, every I_fg^* closed set is an $I_fg_p^\#$ closed set. Therefore $f(L)$ is an $I_fg_p^\#$ closed set in (Y, σ_{if}) . Hence f is an $I_fg_p^\#$ closed mapping.

However, the converse of the above theorem is not always the case. This could be verified by the following example.

Example 3.10 Let (X, τ_{if}) and (Y, σ_{if}) be any two I_f topological spaces where $X = \{q, r\}$, $Y = \{s, t\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, $\sigma_{if} = \{\tilde{0}, B, \tilde{1}\}$, $A = \{< q/0.6, 0.2 >, < r/0.5, 0.5 >\}$ and $B = \{< s/0.3, 0.7 >, < t/0.4, 0.6 >\}$. Let's define an I_f mapping $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ such that $f(q) = s$ and $f(r) = t$. Then an I_f set $L = \{< q/0.2, 0.6 >, < r/0.5, 0.5 >\}$ is an I_f closed set in (X, τ_{if}) , but $f(L)$ is an $I_f g_p^\#$ closed set but not an $I_f g^*$ closed set in (Y, σ_{if}) . Hence, f is an $I_f g_p^\#$ closed mapping but not an $I_f g^*$ closed mapping.

Theorem 3.11 Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an $I_f g^\#$ closed mapping. Then every $I_f g^\#$ closed mapping is an $I_f g_p^\#$ closed mapping.

Proof: Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an $I_f g^*$ closed mapping and let L be an I_f closed set in (X, τ_{if}) . Since f is an $I_f g^\#$ closed mapping, $f(L)$ is an $I_f g^\#$ closed set in (Y, σ_{if}) . Moreover, every $I_f g^\#$ closed set is an $I_f g_p^\#$ closed set. Therefore $f(L)$ is an $I_f g_p^\#$ closed set in (Y, σ_{if}) . Hence f is an $I_f g_p^\#$ closed mapping.

However, the converse of the above theorem is not always the case. This could be verified by the following example.

Example 3.12 Let (X, τ_{if}) and (Y, σ_{if}) be any two I_f topological spaces where $X = \{q, r\}$, $Y = \{s, t\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, $\sigma_{if} = \{\tilde{0}, B, \tilde{1}\}$, $A = \{< q/0.8, 0.2 >, < r/0.7, 0.3 >\}$ and $B = \{< s/0.7, 0.3 >, < t/0.6, 0.4 >\}$. Let's define an I_f mapping $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ such that $f(q) = s$ and $f(r) = t$. Then an I_f set $L = \{< q/0.2, 0.8 >, < r/0.3, 0.7 >\}$ is an I_f closed set in (X, τ_{if}) , but $f(L)$ is an $I_f g_p^\#$ closed set but not an $I_f g^\#$ closed set in (Y, σ_{if}) . Hence, f is an $I_f g_p^\#$ closed mapping but not an $I_f g^\#$ closed mapping.

Theorem 3.13 Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an $I_f g^*p$ closed mapping. Then every $I_f g^*p$ closed mapping is an $I_f g_p^\#$ closed mapping.

Proof: Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an $I_f g^*p$ closed mapping and let L be an I_f closed set in (X, τ_{if}) . Since f is an $I_f g^*p$ closed mapping, $f(L)$ is an $I_f g^*p$ closed set in (Y, σ_{if}) . Moreover, every $I_f g^*p$ closed set is an $I_f g_p^\#$ closed set. Therefore $f(L)$ is an $I_f g_p^\#$ closed set in (Y, σ_{if}) . Hence f is an $I_f g_p^\#$ closed mapping.

However, the converse of the above theorem is not always the case. This could be verified by the following example.

Example 3.14 Let (X, τ_{if}) and (Y, σ_{if}) be any two I_f topological spaces where $X = \{q, r\}$, $Y = \{s, t\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, $\sigma_{if} = \{\tilde{0}, B, \tilde{1}\}$, $A = \{< q/0.2, 0.8 >, < r/0.3, 0.7 >\}$ and $B = \{< s/0.7, 0.3 >, < t/0.6, 0.4 >\}$. Let's define an I_f mapping $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ such that $f(q) = s$ and $f(r) = t$. Then an I_f set $L = \{< q/0.8, 0.2 >, < r/0.7, 0.3 >\}$ is an I_f closed

set in (X, τ_{if}) , but $f(L)$ is an $I_f g_p^\#$ closed set but not an $I_f g^* p$ closed set in (Y, σ_{if}) . Hence, f is an $I_f g_p^\#$ closed mapping but not an $I_f g^* p$ closed mapping.

Theorem 3.15 Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an $I_f g_p^\#$ closed mapping. Then every $I_f g_p^\#$ closed mapping is an $I_f gsp$ closed mapping.

Proof: Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an $I_f g_p^\#$ closed mapping and let L be an I_f closed set in (X, τ_{if}) . Since f is an $I_f g_p^\#$ closed mapping, $f(L)$ is an $I_f g_p^\#$ closed set in (Y, σ_{if}) . Moreover, every $I_f g_p^\#$ closed set is an $I_f gsp$ closed set. Therefore $f(L)$ is an $I_f gsp$ closed set in (Y, σ_{if}) . Hence f is an $I_f gsp$ closed mapping.

However, the converse of the above theorem is not always the case. This could be verified by the following example.

Example 3.16 Let (X, τ_{if}) and (Y, σ_{if}) be any two I_f topological spaces where $X = \{q, r\}$, $Y = \{s, t\}$, $\tau_{if} = \{\emptyset, A, \tilde{1}\}$, $\sigma_{if} = \{\emptyset, B, \tilde{1}\}$, $A = \{\langle q/0.6, 0.4 \rangle, \langle r/0.5, 0.5 \rangle\}$ and $B = \{\langle s/0.3, 0.7 \rangle, \langle t/0.4, 0.6 \rangle\}$. Let's define an I_f mapping $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ such that $f(q) = s$ and $f(r) = t$. Then an I_f set $L = \{\langle q/0.4, 0.6 \rangle, \langle r/0.5, 0.5 \rangle\}$ is an I_f closed set in (X, τ_{if}) , but $f(L)$ is an $I_f gsp$ closed set but not an $I_f g_p^\#$ closed set in (Y, σ_{if}) . Hence, f is an $I_f gsp$ closed mapping but not an $I_f g_p^\#$ closed mapping.

Remark 3.17 Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an I_f mapping. Then $I_f s$ closed mapping, $I_f g^* s$ closed mapping, and $I_f \Psi$ closed mapping are independent to $I_f g_p^\#$ closed mapping. This could be verified with the following example.

Example 3.18 Let (X, τ_{if}) , (Y, σ_{if}) and (Z, η_{if}) be any three $I_f TSS$ s where $X = \{q, r\}$, $Y = \{s, t\}$, $Z = \{u, v\}$, $\tau_{if} = \{\emptyset, A, \tilde{1}\}$, $\sigma_{if} = \{\emptyset, B, \tilde{1}\}$, $\eta_{if} = \{\emptyset, C, \tilde{1}\}$, $A = \{\langle q/0.8, 0.2 \rangle, \langle r/0.7, 0.3 \rangle\}$, $B = \{\langle s/0.3, 0.7 \rangle, \langle t/0.4, 0.6 \rangle\}$ and $C = \{\langle u/0.6, 0.4 \rangle, \langle v/0.5, 0.5 \rangle\}$. Let's define a mapping $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ such that $f(q) = s$ and $f(r) = t$. Then an I_f set $L = \{\langle q/0.2, 0.8 \rangle, \langle r/0.3, 0.7 \rangle\}$ is an I_f closed set in (X, τ_{if}) . Clearly, we observe that $f(L)$ is an $I_f g_p^\#$ closed set but, not an $I_f s$ closed set, an $I_f g^* s$ closed set, and also an $I_f \Psi$ closed set in (Y, σ_{if}) . Let's also define another mapping $g: (Z, \eta_{if}) \rightarrow (Y, \sigma_{if})$ such that $g(u) = s$ and $g(v) = t$. Then an I_f set $D = \{\langle u/0.4, 0.6 \rangle, \langle v/0.5, 0.5 \rangle\}$ is an I_f closed set in (Z, η_{if}) . Clearly, we observe that $g(D)$ is an $I_f s$ closed set, an $I_f g^* s$ closed set, and also an $I_f \Psi$ closed set but not an $I_f g_p^\#$ closed set in (Y, σ_{if}) . Hence, an $I_f s$ closed mapping, an $I_f g^* s$ closed mapping, and an $I_f \Psi$ closed mapping are independent to $I_f g_p^\#$ closed mapping.

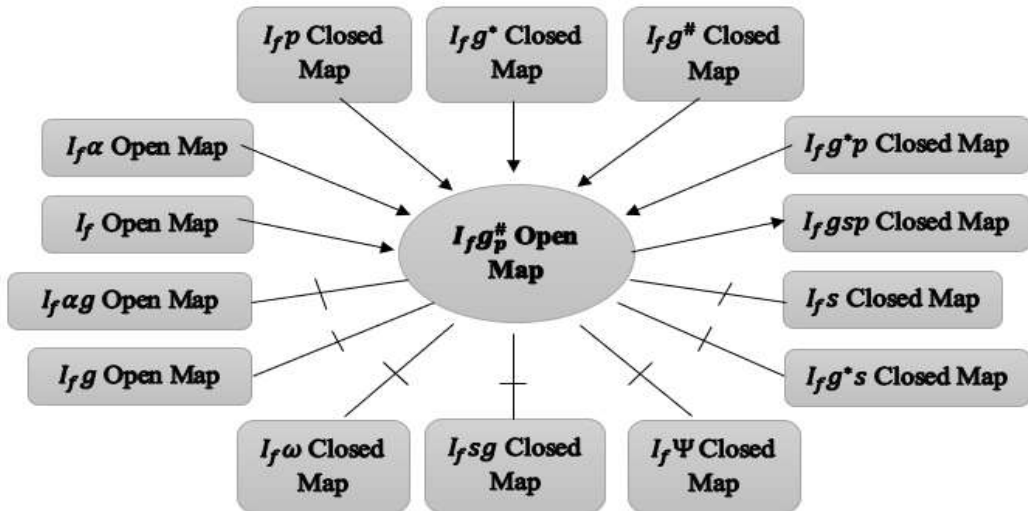
Remark 3.19 Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an I_f mapping. Then $I_f sg$ closed mapping and $I_f \omega$ closed mapping are independent to $I_f g_p^\#$ closed mapping. This could be verified with the following example.

Example 3.20 Let (X, τ_{if}) , (Y, σ_{if}) and (Z, η_{if}) be any three I_f topological spaces where $X = \{q, r\}$, $Y = \{s, t\}$, $Z = \{u, v\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, $\sigma_{if} = \{\tilde{0}, B, \tilde{1}\}$, $\eta_{if} = \{\tilde{0}, C, \tilde{1}\}$, $A = \{< q/0.6, 0.2 >, < r/0.5, 0.5 >\}$, $B = \{< s/0.3, 0.7 >, < t/0.4, 0.6 >\}$ and $C = \{< u/0.6, 0.4 >, < v/0.3, 0.5 >\}$. Let's define a mapping $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ such that $f(q) = s$ and $f(r) = t$. Then an I_f set $L = \{< q/0.2, 0.6 >, < r/0.5, 0.5 >\}$ is an I_f closed set in (X, τ_{if}) . Clearly, we observe that $f(L)$ is an $I_f g_p^\#$ closed set but, not an $I_f sg$ closed set and also an $I_f \omega$ closed set in (Y, σ_{if}) . Let's also define another mapping $g: (Z, \eta_{if}) \rightarrow (Y, \sigma_{if})$ such that $g(u) = s$ and $g(v) = t$. Then an I_f set $D = \{< u/0.4, 0.6 >, < v/0.5, 0.3 >\}$ is an I_f closed set in (Z, η_{if}) . Clearly, we observe that $g(D)$ is an $I_f sg$ closed set and also an $I_f \omega$ closed set but not an $I_f g_p^\#$ closed set in (Y, σ_{if}) . Hence, an $I_f sg$ closed mapping and an $I_f \omega$ closed mapping are independent to an $I_f g_p^\#$ closed mapping.

Remark 3.21 Let $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ be an I_f mapping. Then $I_f g$ closed mapping and $I_f \alpha g$ closed mapping are independent to $I_f g_p^\#$ closed mapping. This could be verified with the following example.

Example 3.22 Let (X, τ_{if}) , (Y, σ_{if}) and (Z, η_{if}) be any three $I_f TSS$ s where $X = \{q, r\}$, $Y = \{s, t\}$, $Z = \{u, v\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, $\sigma_{if} = \{\tilde{0}, B, \tilde{1}\}$, $\eta_{if} = \{\tilde{0}, C, \tilde{1}\}$, $A = \{< q/0.8, 0.2 >, < r/0.7, 0.3 >\}$, $B = \{< s/0.3, 0.7 >, < t/0.4, 0.6 >\}$ and $C = \{< u/0.6, 0.4 >, < v/0.3, 0.5 >\}$. Let's define a mapping $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ such that $f(q) = s$ and $f(r) = t$. Then an I_f set $L = \{< q/0.2, 0.8 >, < r/0.3, 0.7 >\}$ is an $I_f C$ set in (X, τ_{if}) . Clearly, we observe that $f(L)$ is an $I_f g_p^\#$ closed set but, not an $I_f g$ closed set and also an $I_f \alpha g$ closed set in (Y, σ_{if}) . Let's also define another mapping $g: (Z, \eta_{if}) \rightarrow (Y, \sigma_{if})$ such that $g(u) = s$ and $g(v) = t$. Then an I_f set $D = \{< u/0.4, 0.6 >, < v/0.5, 0.3 >\}$ is an $I_f C$ set in (Z, η_{if}) . Clearly, we observe that $g(D)$ is an $I_f g$ closed set and also an $I_f \alpha g$ closed set but not an $I_f g_p^\#$ closed set in (Y, σ_{if}) . Hence, an $I_f g$ closed mapping and an $I_f \alpha g$ closed mapping are independent to an $I_f g_p^\#$ closed mapping.

Remark 3.23 The following diagram represents the relations of $I_f g_p^\#$ closed mappings with existing I_f mappings.



4. Characterizations of $I_f g_p^\#$ Closed Mappings

Theorem 4.1 An I_f mapping $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ is an $I_f g_p^\#$ closed mapping if and only if $I_f cl(f(L)) \subseteq f(I_f cl(L))$, for every I_f subset L of (X, τ_{if}) .

Proof: Necessary Condition:- Suppose f is an $I_f g_p^\#$ closed mapping. Since for every I_f subset L of (X, τ_{if}) , $I_f cl(L)$ is an I_f closed set in (X, τ_{if}) then $f(I_f cl(L))$ is an $I_f g_p^\#$ closed set in (Y, σ_{if}) . As $L \subseteq I_f cl(L)$, $f(L) \subseteq f(I_f cl(L))$ which implies $I_f cl(f(L)) \subseteq I_f cl(f(I_f cl(L)))$. Hence $I_f cl(f(L)) \subseteq f(I_f cl(L))$.

Sufficient Condition:- Let L be an I_f closed set in (X, τ_{if}) . Since $I_f cl(f(L))$ is the smallest $I_f g_p^\#$ closed set containing $f(L)$, $f(L) \subseteq I_f cl(f(L)) \subseteq f(I_f cl(L)) = f(L)$ which implies $f(L) = I_f cl(f(L))$. Therefore, $f(L)$ is an $I_f g_p^\#$ closed set in (Y, σ_{if}) . Hence, f is an $I_f g_p^\#$ closed mapping.

Theorem 4.2 Composition of any two $I_f g_p^\#$ closed mappings need not be an $I_f g_p^\#$ closed mapping. This could be verified with the following example.

Example 4.3 Let (X, τ_{if}) , (Y, σ_{if}) , and (Z, η_{if}) be any three I_f topological spaces, where $X = \{q, r\}$, $Y = \{s, t\}$, $Z = \{u, v\}$, $\tau_{if} = \{\tilde{0}, A, \tilde{1}\}$, $\sigma_{if} = \{\tilde{0}, B, \tilde{1}\}$, $\eta_{if} = \{\tilde{0}, C, \tilde{1}\}$, $A = \{<$

$q/0.6, 0.4 >, < r/0.5, 0.5 >\}$, $B = \{< q/0.8, 0.2 >, < r/0.7, 0.3 >\}$, and $C = \{< q/0.3, 0.7 >, < r/0.4, 0.6 >\}$. Let's define an I_f mapping $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ such that $f(q) = s$, and $f(r) = t$. Then an I_f set $L = \{< p/0.4, 0.6 >, < q/0.5, 0.5 >\}$ is an I_f closed set in (X, τ_{if}) and $f(L)$ is an $I_f g_p^\#$ closed set in (Y, σ_{if}) . Then f is an $I_f g_p^\#$ closed mapping. Let's also define another I_f mapping $g: (Y, \sigma_{if}) \rightarrow (Z, \eta_{if})$ such that $g(s) = u$, and $g(t) = v$. Then an I_f set $D = \{< s/0.2, 0.8 >, < t/0.3, 0.7 >\}$ is an I_f closed set in (Y, σ_{if}) and $g(D)$ is an $I_f g_p^\#$ closed set in (Z, η_{if}) . Then g is an $I_f g_p^\#$ closed mapping. However, $(g \circ f)(L)$ is not an $I_f g_p^\#$ closed set in (Z, η_{if}) . Hence $g \circ f: (X, \tau_{if}) \rightarrow (Z, \eta_{if})$ is not an $I_f g_p^\#$ closed mapping.

Theorem 4.4 Let (X, τ_{if}) , (Y, σ_{if}) , and (Z, η_{if}) be any three I_f topological spaces where $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ is an I_f closed mapping and $g: (Y, \sigma_{if}) \rightarrow (Z, \eta_{if})$ is an $I_f g_p^\#$ closed mapping then $g \circ f: (X, \tau_{if}) \rightarrow (Z, \eta_{if})$ is an $I_f g_p^\#$ closed mapping.

Proof: Let L be an I_f closed set in (X, τ_{if}) . Then $f(L)$ is an I_f closed set in (Y, σ_{if}) . Since $g: (Y, \sigma_{if}) \rightarrow (Z, \eta_{if})$ is an $I_f g_p^\#$ closed mapping, $g(f(L)) = (g \circ f)(L)$ is $I_f g_p^\#$ closed set in (Z, η_{if}) . Hence, $g \circ f: (X, \tau_{if}) \rightarrow (Z, \eta_{if})$ is an $I_f g_p^\#$ closed mapping.

Theorem 4.5 Let (X, τ_{if}) , (Y, σ_{if}) , and (Z, η_{if}) be any three I_f topological spaces and let $g \circ f: (X, \tau_{if}) \rightarrow (Z, \eta_{if})$ be a composite function where $f: (X, \tau_{if}) \rightarrow (Y, \sigma_{if})$ and $g: (Y, \sigma_{if}) \rightarrow (Z, \eta_{if})$ are I_f functions, then

- (i) g is an $I_f g_p^\#$ closed mapping, if $g \circ f$ is an $I_f g_p^\#$ closed mapping and f is both an I_f continuous and surjective function.
- (ii) f is an $I_f g_p^\#$ closed mapping, if $g \circ f$ is an $I_f g_p^\#$ closed mapping and g is both an $I_f g_p^\#$ irresolute and injective function.
- (iii) g is an $I_f g_p^\#$ closed mapping, if $g \circ f$ is an $I_f g_p^\#$ closed mapping and f is both $I_f g_p^\#$ continuous and surjective function.

Proof: (i) Let f be both an I_f continuous and surjective function and let L be an I_f closed set in (Y, σ_{if}) . Then $f^{-1}(L)$ is an I_f closed set in (X, τ_{if}) . It is given that $g \circ f$ is an $I_f g_p^\#$ closed mapping. Then $(g \circ f)(f^{-1}(L))$ is an $I_f g_p^\#$ closed set in (Z, η_{if}) which implies $g(L)$ is an $I_f g_p^\#$ closed set in (Z, η_{if}) for every I_f closed set L in (Y, σ_{if}) . Hence g is an $I_f g_p^\#$ closed mapping.

(ii) Let L be an I_f closed set in (X, τ_{if}) . Then $(g \circ f)(L)$ is an $I_f g_p^\#$ closed set in (Z, η_{if}) and since g is an $I_f g_p^\#$ irresolute and injective function, $g^{-1}((g \circ f)(L)) = f(L)$ is an $I_f g_p^\#$ closed set in (Y, σ_{if}) . Hence, f is an $I_f g_p^\#$ closed mapping.

(iii) Let L be an I_f closed set in (Y, σ_{if}) . Then $f^{-1}(L)$ is an $I_f g_p^\#$ closed set in (X, τ_{if}) and since f is both an $I_f g_p^\#$ continuous and surjective function, $(g \circ f)(f^{-1}(L)) = g(L)$ which is an $I_f g_p^\#$ closed set in (Z, η_{if}) . Hence g is $I_f g_p^\#$ closed mapping.

REFERENCES

- [1] Abhirami .S and Dhavaseelan .R, 2014, Intuitionistic Fuzzy $g^{\#}$ Closed Sets, International Journal of Research in Advent Technology, 2(3).
- [2] Arun Prakash .K and Santhi .R, (2012), Intuitionistic fuzzy semi generalized closed mappings, International Journal of Mathematics and soft computing, 2(2), pp. 85-94.
- [3] Atanassov .K and Stoeva .S, 1983, Intuitionistic Fuzzy Sets, In Polish Symposium on Interval and Fuzzy Mathematics, Poznam, pp. 23-26.
- [4] Bajpai .J.P and Thakur .S.S, 2016, Intuitionistic Fuzzy g^* - Closed Sets, International Journal of Innovative Research in Science and Engineering, 2, pp. 19-30.
- [5] Bajpai .J.P and Thakur .S.S, 2018, Intuitionistic Fuzzy g^*p - Closed Sets, Global Journal of Pure and Applied Mathematics, 14 (17): 955-975.
- [6] Bajpai .J.P and Thakur .S.S, 2017, Intuitionistic Fuzzy sgp Closed Set, International Journal of Latest Trends in Engineering and Technology, 8(1), pp. 636-642.
- [7] Chang .C.L, 1968, Fuzzy Topological Spaces, J. Math. Anal. Appl. 24, pp. 182-190.
- [8] Coker .D and Demirci .M, 1995, On Intuitionistic Fuzzy Points, Notes on Intuitionistic Fuzzy Sets, pp. 79-83.
- [9] Coker .D, 1997, An Introduction to Intuitionistic Fuzzy Topological Spaces, Fuzzy Sets and Systems, 88, pp. 81-89.
- [10] Coskun .E and Coker .D, 1998, On Neighbourhood Structures in Intuitionistic Fuzzy Topological Space, Mathematica Balkanica, 12(3-4), pp. 289-293.
- [11] Gowri .C.S, Sakthivel .K, and Kalamani .D, (2014), Generalized alpha closed mappings and open mappings in intuitionistic fuzzy topological spaces, International Journal of Mathematical Archive, 5(1), pp. 240-247.
- [12] Gurcay .H, Coker .D, and Es. A. Haydar, 1997, On Fuzzy Continuity in Intuitionistic Fuzzy Topological Spaces, Journal of Fuzzy Mathematics, 5, pp. 365-378.
- [13] Kalamani .D, Sakthivel .K, and Gowri .C.S, 2012, Generalized Alpha Closed Sets in Intuitionistic Fuzzy Topological Spaces, Applied Mathematical Sciences, 6 (94), pp. 4691-4700.
- [14] Noiri .T, 1973, A Generalization of Closed Mappings, Atti. Acad. Naz. Lincei Rend. Cl. Ser. Fis. Mat. Natur., 54, pp. 412-415.
- [15] Parimala .M, Indirani .C, and Selvakumar .A, 1998, On Intuitionistic Fuzzy Ψ -Closed Sets and its Application, Annals of Fuzzy Mathematics and Informatics J. Contemp. Math. Sciences, 10(1), pp. 77-85.
- [16] Pious Missier .S and Babisha Julit .R.L, 2021, Intuitionistic Fuzzy g^*s -Closed Sets, International journal of Mathematical Archive, 12(2), pp. 164-173.
- [17] Pious Missier .S and Gabriel Raja .S, 2022, A new class of closed sets in intuitionistic fuzzy topological spaces, IJFANS, 11(13), pp. 3410-3420.
- [18] Rajarajeswari .P and Senthil Kumar .L, 2011, Generalized Pre-Closed Sets in Intuitionistic Fuzzy Topological Spaces, International Journal of Fuzzy Mathematics and Systems, 3, pp. 324-328.
- [19] Sakthivel .K, 2012, Intuitionistic Fuzzy Alpha Generalized Closed Sets and Intuitionistic Fuzzy Alpha Generalized Open Sets, The Mathematical Education 4.
- [20] Santhi .R and Arun Prakash .K, 2010, On Intuitionistic Fuzzy Semi-Generalized Closed Sets and its Applications, Int. J. Contemp. Math. Sciences, 5, pp. 1677-1688.
- [21] Santhi .R and Jyanthi .D, 2009, Intuitionistic Fuzzy Generalized Semi Pre Closed Sets, Tripura Math. Soci., pp 61-72.
- [22] Thakur .S.S, and Rekha Chaturvedi, 2008, Generalized Closed Set in Intuitionistic Fuzzy Topology, The Journal of Fuzzy Mathematics, 16(3), pp. 559-572.

- [23] Thakur S.S. and Bajpai .J.P, 2010, Intuitionistic Fuzzy ω -Closed Sets and Intuitionistic Fuzzy ω -Continuity, International Journal of Contemporary Advanced Mathematics, 1(1), pp. 1-15.
- [24] Thakur S.S. and Bajpai .J.P, 2011, Semi Generalized Closed Sets in Intuitionistic Fuzzy Topology, International Review of Fuzzy Mathematics, 6(2), pp. 69-76.
- [25] Young Bae Junn and Seok Zun Song, 2005, Intuitionistic Fuzzy Semi-Pre-Open Sets and Intuitionistic Semi-Pre-Continuous Mappings, J. Appl. Math. & Computing, pp. 467-474.
- [26] Zadeh .L.H, 1965, Fuzzy Sets, Information and Control, 18 pp. 338-353.