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Performance Analysis Of Music And ESPRIT Algorithms For Direction-Of-Arrival Estimation On Coprime Arrays

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Direction-of-Arrival (DOA) estimation is essential in array signal processing, aiding in pinpointing signal sources in systems such as radar and wireless communication networks. Coprime arrays have attracted interest in this area because they provide additional degrees of freedom (DOFs) while maintaining low hardware requirements due to their sparse sensor configurations. Over time, many algorithms have been developed to enhance these arrays, including popular subspace-based methods like MUSIC and ESPRIT. A comprehensive spectral search is necessary to achieve MUSIC's renowned precision, but doing so demands additional processing power. In contrast, ESPRIT offers a more efficient, closed-form solution by utilizing the rotational invariance property of the coarray structure. In this study, we compare ESPRIT and MUSIC for DOA estimation using coprime arrays, highlighting their accuracy and processing efficiency under different conditions.

Index Terms—Direction of Arrival (DOA) Estimation, Co- prime Array, Virtual Array Interpolation, FBSS.

I. INTRODUCTION

Accurate localization of signal sources is important in modern technological systems, making Direction-of-Arrival (DOA) estimation important in array signal processing, which involves finding the direction from which a signal arrives on an array of sensors. It plays a crucial role in several modern technological applications such as radar systems, sonar, wire- less communications, satellite tracking, navigation systems, radio astronomy, and speech signal enhancement. With the exponential rise in wireless data transmission, accurate and high-resolution DOA estimation techniques have become nec- essary for resource allocation, target tracking, beamforming, and interference mitigation.

There are various kinds of array structures, among which the Uniform Linear Array (ULA) is the most widely used due to its simplicity and ease of implementation. A ULA consists of sensors placed at uniform spacing, typically half the signal wavelength, to avoid spatial aliasing [5]. However, ULAs are limited in terms of the number of sources they can resolve, which is generally less than or equal to the number of sensors. To achieve a high spatial resolution, ULA requires a large number of sensors, which leads to increased hardware costs and complexity. These drawbacks motivate the use of sparse array configurations.

Coprime array is one having these sparse array configuration, which significantly enhances the degrees of freedom (DOFs) while using fewer physical sensors [7]. A coprime array is formed by combining two subarrays with sensor spacings that are integer multiples of a basic unit and are coprime to each other. For instance, if M and N are coprime integers, then one subarray places sensors at multiples of Nd and the other at multiples of Md. The resulting sensor positions create a non-uniform linear array with a larger virtual aperture, which enables the resolution of more sources than physical sensors [3] [4]. The coprime structure allows the formation of a virtual uniform linear array (coarray) with a dense set of lags, which is essential for high-resolution DOA estimation.

The Moving Coprime Array (MCA) builds upon this conceptby incorporating array movement along a predetermined usually linear path, which enhances both the effective aperture and the quantity of distinct virtual lags. An augmented virtual array with greater DOFs and less spatial aliasing is created by gathering data at various time points during motion. The MCA is especially helpful in situations where there are few snapshots or coherent sources.

Various algorithms have been proposed to estimate DOA using such arrays. These include classical beamforming, subspace-based methods like MUSIC (Multiple Signal Classification). ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques), and optimization-based methods like sparse reconstruction and nuclear norm minimization. Among them MUSIC and ESPRIT stand out for their high-resolution capabilities and reliability under different scenarios [20].

The music algorithm is a subspace-bases technique that estimates DOAs by taking advantage of the orthogonality between the noise subspace and the signal subspace. Considering the coprime array, the received signal is first modelled, and the covariance matrix is formed from multiple time snapshots. For coherent signals, the spatial smoothing technique known as Forward/Backward Spatial Smoothing (FBSS) is used to restore the rank of the covariance matrix [1]. Additionally, Nuclear Norm Minimization (NNM) is used to interpolate missing elements in the virtual array, thereby enabling the formation of a complete Toeplitz matrix. The virtual coarray is transformed into a filled uniform array by interpolation, and the MUSIC spectrum is computed by scanning the angular domain to locate peaks, which correspond to the estimated DOAs. The algorithm offers very high resolution but it is computationally intensive due to its grid search mechanism.

On the other hand, ESPRIT offers a more efficient closed form solution by applying the rotational invariance property inherent in subarrays of the virtual ULA (Uniform Linear Array). For coprime arrays, the received data is vectorized and mapped into a coarray. After interpolation using NNM, the virtual ULA is partitioned into two overlapping subarrays with a known displacement. The signal subspaces corresponding to these subarrays are extracted, and a rotational operator is computed using least squares. The eigenvalues of this operator are then used to directly compute the DOAs. ESPRIT avoids spectral search and has lower computational complexity but typically performs better when the signal are uncorrelated.

This research compares the performance of MUSIC and ESPRIT techniques by estimating signal directions utilizing coprime arrays and evaluates their performance by determining how much their predictions differ from the actual angles(using RMSE) under different circumstances, including changing noise levels (SNR) and the quantities of data (snapshots). For MUSIC, methods such as FBSS and matrix completion are used to tackle difficult scenarios where signals intersect (coherent signals), while ESPRIT uses a coarray configuration to aid in computations [1] [2]. The findings indicate the optimal conditions for each approach, like MUSIC excels with overlapping signals, while ESPRIT offers improved speed and precision for non-overlapping signals. This helping users in selecting appropriate choice according to their requirements such as managing noise, signal types or time limitations.

II. SIGNAL MODEL

For a static coprime array with M + N-1 physical sensors, the received signal at time t, under K far-field narrowband signals from directions $\theta = [\theta_{1,}\theta_2 \dots . \theta_K]^T$, is modeled as,

$$x_S(t) = s(t)\sum_{k=1}^{K} \alpha_k a_S(\theta_k) + n_S(t)$$

This equation shows the signal model for a coprime array that is receiving multiple signals from different directions. It describes how the total signal received at the array is formed by combining contributions from all incoming sources [16]. Each signal is affected by its direction of arrival and travels through the environment before reaching the sensors. This model assumes that signals are narrowband and coming from far-field sources, meaning the wavefronts can be treated as plane waves across the array.
Where $x_S(t) \in C^{|S| \times 1}$is the received signal vector, s(t) is the reference waveform, a_k represents the complex fading coefficient of the k-signal, and $n_s(t) \sim CN(0, \sigma_n^2 I)$ is additive white gaussian noise with power σ_n^2. The steering vector is defined as:

$$a_S(\theta_k) = \left[e^{-l_{|S|}\underline{\theta_k}}, e^{-l_{|S|-1}\underline{\theta_k}}, \dots, e^{-l_1\underline{\theta_k}}\right]^T,$$

Combines the phase shifts at each sensor position $l_i \in S$, where $l_1 = 0$, denotes the reference sensor. This steering vector plays a key role in modeling how a signal arriving from direction θ_k appears across the spatial positions of the array. Each term in the vector represents the phase shift induces by the corresponding sensor location due to the signal's arrival angle. The sensor at the reference location l_1
sees the signal without delay, while sensors at other positions observe phase shifts that grow with distance. This phase relationship forms a spatial pattern that is unique to each direction, which is later used to estimate the DOA. The term

$$\underline{\theta_k} = \frac{j2\pi \sin \sin (\theta_k)}{\lambda}$$

This equation normalizes the DOA into a spatial frequency, with λ as the signal wavelength. The exponential term quantifies the phase delay at the i-th sensor due to the k-th signal's angle of arrival. The normalized spatial frequency $\underline{\theta_k}$allows the steering vector to be expressed in exponential form, which simplifies the mathematical analysis of phase delays across the array. By representing direction in terms of spatial frequency, standard array processing techniques such as Fourier-based and subspace-based algorithms can be more easily applied. This abstraction also helps in generalizing the model across various wavelength and sensor spacings without modifying the core formulation. The manifold matrix A_S, aggregating all steering vectors is given as,

$$A_S = [a_S(\theta_1), a_S(\theta_2), \dots, a_S(\theta_K)],$$

Serves as the foundation for subspace-based DOA estimation algorithms. The structure of the manifold matrix A_S provides a compact and complete description of how all Ksources are received across the array aperture. Its columns span the signal subspace, which is separated from the noise subspace using techniques like eigenvalue decomposition. This separation is crucial for accurate direction finding, especially in scenarios where the number of sources is close to or exceeds the number of physical sensors as in coprime arrays. This model enables high-resolution angle estimation by leveraging the coprime array's sparse geometry while addressing challenges like coherent signals through preprocessing techniques [4].

III. MUSIC ALGORITHM ON MOVING COPRIME ARRAYS

The MUSIC (Multiple Signal Classification) algorithm has long been recognized for its accuracy and ability to resolve closely spaces signals in high-resolution direction of arrival (DOA) estimation but conventional MUSIC implementations frequently suffer from limitations in the quantity of physical sensors and from their inability to manage coherent signals without using decorrelation methods. Coprime arrays are utilized to create a virtual array with higher aperture and degrees of freedom in order to overcome these limitations which results in higher resolution with fewer physical components. The rank of the observed signal space also may be improved by introducing motion to these arrays that allows for the capture of snapshots with temporal diversity. The MUSIC method can be successfully adapted to moving coprime arrays by combining these ideas, allowing for accurate DOA estimation even when these is signal coherence and sparse sampling.

The MUSIC algorithm leverages the extended degrees of freedom (DOFs) of the coprime array through virtual coarray processing. For a moving coprime array (MCA), the covariance matrix is constructed from snapshots collected at multiple time instantst$_i$ [7]. Let $x_{ms}(t_i)$ denote the signal received at time t_i modeled as:

$$x_{mS}(t_i) = s(t_i)B_{mS}\alpha + n_S(t_i),$$

Where $B_{mS} = \left[e^{l_i\delta_1 a_S}(\theta_1), \dots, e^{l_i\delta_K a_S}(\theta_K)\right]$ incorporates motion-induced phase shifts $l_i = vt_i$ and $\delta_k = j2\pi \sin\sin\theta_k /\lambda$[8]. The covariance matrix R_{ms} is derived as:

$$R_{mS} = A_S\Phi A_S^H,$$

Where $\Phi = \mathrm{diag}(\phi_1, \dots, \phi_K)$ contains source powers and As is the manifold matrix of the static coprime array.

To resolve coherent signals, vectorization maps R_{ms} to a virtual array:

$$y_v = vec(R_{mS}) = A_v p$$

Where$A_v = A_S^* \odot A_S$ spans the virtual sensors $V = \{l_{i2} - l_{i1} \mid i1, i2 = 1,2, \dots, |S|\}$. The discontinuous virtual array V is interpolated into a hole-free uniform linear array (ULA) I with sensor positions $l_w \in [(V), (V)]$[5]. Missing virtual lags are filled via nuclear norm minimization (NNM) [3] [13]

$$\frac{1}{2}|\hat{F} \circ B - F|_F^2 + \mu|\hat{F}|_*,$$

Where F is the Toeplitz matrix of the incomplete data y_I, B is a binary mask, and μ balances data fidelity and low rank constraints.
The restored Toeplitz matrix $\hat{F}$ is rearranged into a full rank ULA signal $\hat{y}_I$. Forward/backward spatial smoothing (FBSS) is applied to $\hat{y}_I$ to decorrelate coherent sources:

$$R^{fb} = \frac{1}{2Q}\sum_{q=1}^{Q} \left(\hat{y}_I^q(\hat{y}_I^q)^H + \Pi(\hat{y}_I^q)^*(\hat{y}_I^q)^T\Pi^H\right),$$

Where $\Pi$is the exchange matrix. Eigen-decomposing R^{fb}seperates the noise subspace U_N. The MUSIC spatial spectrum is computed as:

$$P(\theta) = \frac{1}{a^H(\theta)U_N U_N^H a(\theta)},$$

Where $a(\theta)$is the steering vector of the interpolated ULA. Peaks in $P(\theta)$ correspond to estimated DOAs. The spatial spectrum is evaluated over a predefined angular grid that spans over the entire field of view. Each point on the grid corresponds to a possible signal arrival angle. For each angleθ, the corresponding steering vector $a(\theta)$ is projected onto the noise subspace U_N. Since true signal directions lie in the signal subspace and are orthogonal to the noise subspace, the projection yields minimal energy at these directions. The MUSIC algorithm identifies the DOAs by locating the angles θ at which the denominator $a^H(\theta)U_N U_N^H a(\theta)$ is minimized, resulting in sharp peaks in the spectrum.
This grid-based search method allows the estimation of multiple DOAs simultaneously even when signals are closely spaced or partially coherent, provided that the spatial smoothing and matrix completion steps are effectively applied. Even if this method is computationally intensive, this exhaustive search over the angle grid ensured high-resolution performance which makes MUSIC suitable for scenarios where accuracy in angular discrimination is critical.

IV. ESPRIT ALGORITHM IN COPRIME ARRAYS

While the MUSIC algorithm relies on exhaustive grid search over a defined angular spectrum. It can suffer from high computational cost and sensitivity to overcome mismatches in the estimation of direction accurately, especially when the true direction of arrival (DOAs) do not align with the sampling grid. To overcome these limitations subspace-based techniques such as Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) have been proposed. ESPRIT eliminates the need for spectral search by directly utilizing the rotational invariance between subarrays with identical structures [11]. When applied in the coarray domain of a coprime array, ESPRIT can extract this invariance from virtually generated shift-invariant subarrays, enabling high-resolution and off-grid DOA estimation with reduced complexity [14]. This section presents the coarray domain formation of the ESPRIT algorithm considering coprime arrays.

The received signal covariance matrix R_{xx} for the static coprime array is computed as:

$$R_{xx} = E[x_S(t)x_S^H(t)] = A_S \Phi A_S^H + \sigma_n^2 I$$

Where $\Phi = diag(\phi_1, \dots, \phi_K)$ contains source powers. Vectorizing /// generates the coarray signal:

$$y_v = vec(R_{xx}) = A_v p,$$

Where $A_v = A_S^* \odot A_S$ spans the virtual sensors

$$V = \{l_{i_2} - l_{i_1} \mid l_{i_1}, l_{i_2} \in S\}.$$

The virtual coarray$V$ is interpolated into a uniform linear array (ULA) I using nuclear norm minimization (NNM) to fill missing lags, yielding the completed covariance matrix [15]. The interpolated ULA I is partitioned into two overlapping subarrays S_X with a displacement $\Delta d = d$

$$S_X = \{0, d, \dots, (L-1)d\}, \quad S_Y = \{d, 2d, \dots, Ld\},$$

Where $L = |I| - 1$. The corresponding manifold matrices$(A_X) and (A_Y)$ satisfy

$$A_Y = A_X \Phi$$

with:

$$\Phi = diag\left(e^{-j2\pi\Delta d sinsin\,\theta_1/\lambda}, \dots, e^{-j2\pi\Delta d sinsin\,\theta_K/\lambda}\right)$$

The spatially smoothed covariance matrix //// is eigen decomposed:

$$\widehat{R_{ss}} = U_S \Lambda_S U_S^H + U_N \Lambda_N U_N^H,$$

WhereU_S(signal subspace) and U_N(noise subspace) are orthogonal. The signal subspaces $U_{S,X}$ *and* ($U_{S,Y}$ for S_X *and* S_Y are extracted by selecting rows corresponding to each subarray:

$$U_{S,X} = U_S(1{:}L, :), \quad U_{S,Y} = U_S(2{:}L+1, :)$$

The rotational invariance relationship is:

$$U_{S,Y} = U_{S,X}\Psi$$

where $\Psi$is the rotational operator. The eigenvalues $\psi_k\}_{k=1}^{K}$ of the estimated rotational operator Ψ directly encode the angular information of the sources. The DOAs are obtained in closed form as

$$\theta_k = arcsin\, arcsin \left(-\frac{\lambda}{2\pi\Delta d} \cdot I(ln\, ln\, \psi_k) \right).$$

Where Δd denoted the known inter-element spacing between the shift invariant subarrays. This method avoids the need for grid search or spatial spectrum evaluations which was required in MUSIC-based techniques. The ESPRIT algorithm is implemented in the coarray domain and utilizes the full-rank coarray covariance matrix derived from second-order signal statistics and exploits the rotational invariance between two overlapping virtual ULAs [11]. BY performing eigen decomposition of the signal subspace and solving a generalized eigenvalue problem, the algorithm efficiently estimates multiple off-grid DOAs. The approach benefits from the enhanced aperture and degrees-of-freedom which improves the efficiency of the estimated directions.

V. SIMULATION RESULTS

The performance of MUSIC and ESPRIT algorithms for Direction of Arrival (DOA) Estimation on coprime array is evaluated through Monte Carlo simulations under varying SNR levels and snapshot counts using CVX toolbox [9]. The coprime array configurations include a moving coprime array (MCA) for MUSIC$(M = 2, N = 5)$ with physical sensors at $\{-8d, -6d, -4d, -3d, -2d, 0\}$ and a static coprime array for ESPRIT $(M = 3, N = 5)$ with sensors at $\{0, 3d, 5d, 6d, 9d, 10d, 12d, 15d, 20d, 25d\}$ where $d = \lambda/2$. MUSIC processes five coherent signals at $\theta = \{0°, 10°, 20°, 30°, 40°\}$ using nuclear norm minimization (NNM) and forward/backward spatial smoothing (FBSS), while ESPRIT resolves two uncorrelated signals at $\theta = \{-10°, 15°\}$ via coarray interpolation and rotational invariance.

The RMSE is computed for MUSIC as:

$$RMSE = \sqrt{\frac{1}{200K} \sum_{o=1}^{200} \sum_{k=1}^{K} \left(\widehat{\theta_k}(o) - \theta_k\right)^2},$$

And for ESPRIT as:

$$RMSE = \sqrt{\frac{1}{QK} \sum_{q=1}^{Q} \sum_{k=1}^{K} \left(\widehat{\theta_{k,q}} - \theta_k\right)^2}.$$

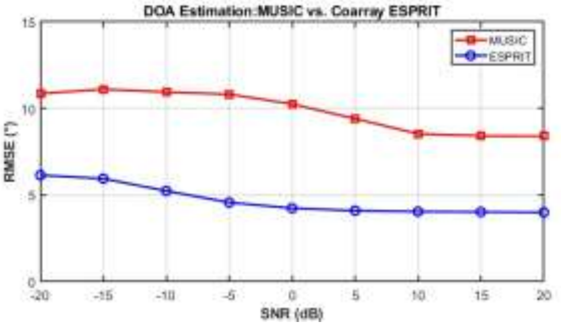

Fig.1 RMSE vs SNR for MUSIC and ESPRIT

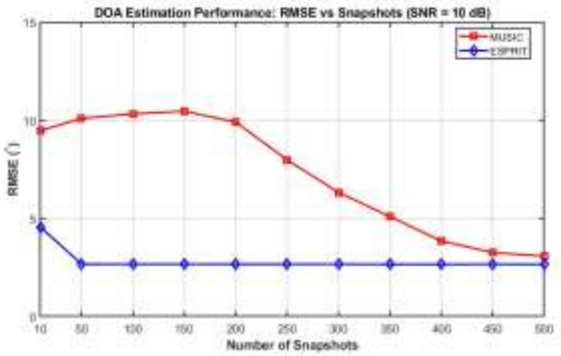

Fig.2 RMSE vs Number of Snapshots (SNR = 10dB)

For RMSE vs SNR (Fig.1), ESPRIT achieves superior performance compared to MUSIC, especially as the SNR increases. Specifically, ESPRIT stabilizes at a RMSE of approximately $4.00°$ for $SNR \geq 10\,Db$. This indicates that ESPRIT provides consistent and accurate Direction of Arrival Estimation even in moderately noisy environments. The reason behind this strong performance lies in ESPRIT's closed form solution, which directly computes the DOAs without requiring a spectral search over the angle domain. This characteristic significantly reduces the computational complexity, making ESPRIT highly efficient for real-time or low latency applications

In contrast, MUSIC's performance becomes stable at an RMSE of about $8.40°$, primarily due to residual errors introduced during the matric completion process. MUSIC requires a grid-based spectral search, where the spatial spectrum is scanned to detect peaks corresponding to source directions. This not only increases computational cost but also introduces approximation errors due to finite grid resolution. Additionally, MUSIC relies heavily on preprocessing techniques such as forward/backward spatial smoothing (FBSS) and matrix completion to handle coherent sources. While these techniques enable MUSIC to work in more complex signal environments, they also introduce estimation variance, particularly at lower SNRs where the signal components are weaker and more susceptible to noise.

On the other hand, ESPRIT leverages the rotational invariance property of the virtual coarray structure, allowing it to form a shift-invariant pair of subarrays. This structural advantage simplifies the estimation process and makes ESPRIT robust against noise in uncorrelated signal scenarios, hence achieving better performance under high SNR conditions with fewer snapshots and less computational burden.
For RMSE vs snapshots (Fig.2), ESPRIT maintains stable accuracy $2.65°$ to $3.44°$ even at $L = 10$ snapshots, while MUSIC requires $L \geq 300$ to achieve comparable precision $3.07°$ at $L = 500$. MUSIC's erratic performance below $L = 300$ from insufficient data for covariance matrix completion [10], whereas ESPRIT leverages the full-rank coarray covariance matrix for robust estimation.

CONCLUSION

This study presents an in- depth comparative analysis of the MUSIC and ESPRIT algorithms for Direction of Arrival (DOA) Estimation using coprime arrays [19]. The performance was assessed across varying conditions of signal to noise ratio (SNR) and number of snapshots and the simulation results clearly indicate that ESPRIT significantly outperforms MUSIC for uncorrelated sources, particularly in scenarios with high SNR. Specifically, as shown in Figure 1, ESPRIT achieves a Root Mean Square Error (RMSE) as low as 4.00 for $SNR \geq 10$, maintaining accuracy with minimal computational effort. However, the limitations of ESPRIT become evident when handling coherent sources, such as those arising from multipath propagation or closely spaced angles [18]. In such cases, MUSIC demonstrates more accuracy due to its forward/backward spatial smoothing (FBSS) and matrix completion techniques that help decorrelate sources. Figure 2 shows that MUSIC achieves comparable precision ($RMSE \approx 3.07$) only when the number of snapshots exceeds 300, reflecting its higher sensitivity to data volume and noise. Even at high SNR, MUSIC exhibits a high RMSE of around 8.4 mainly due to imperfections in matrix completion under practical conditions.
Hence, the trade-off between computational efficiency and capability to resolve coherent signals becomes apparent. ESPRIT is preferable in low-latency environments and when signals are uncorrelated, due to its simplicity and speed. MUSIC, though computationally heavier it makes this technique useful in scenarios involving coherence, such as urban multipath environments or congested spectrum bands [6].

FUTURE SCOPE

This comparative study Highlights the strengths and limitations of both MUSIC and ESPRIT algorithms. However, there are several ways future research can improve these methods to make them useful and practical in real-world scenarios
One promising method is to develop hybrid algorithms that combine the fast, closed-form processing of ESPRIT with the ability of MUSIC to handle coherent signals (like multipath or overlapping sources). For example, ESPRIT can be enhanced with preprocessing techniques such as Forward/Backward Spatial Smoothing (FBSS) or matrix completion [17]. This would allow ESPRIT to work better with coherent signals, without needing the complex spectral search used by MUSIC.
Another important area is to adapt these algorithms for real time and dynamic environments. In actual radar and wireless systems, signals come from moving sources and experience time-varying changes due to noise, fading or reflections. Adding features like motion tracking,

adaptive interpolation would allow MUSIC and ESPRIT to perform well in practical systems like automotive radars, 5G/6G base stations, or indoor tracking systems.
Machine learning also offers new possibilities. Using deep learning models, it is possible to estimate missing elements in the coarray, decide the best smoothing parameters or automatically detect when signals are coherent. A smart system could then choose the most suitable algorithm (MUSIC or ESPRIT) or switch between them as needed, on the real time data it receives.
In addition, future work should test these algorithms on other types on arrays beyond coprime arrays. This includes nested arrays, minimum redundancy arrays or custom geometries used in practice. Creating a standard benchmarking system to compare different algorithms across factors like accuracy (RMSE), resolution, computation time and adaptability world be very valuable
Finally, it's important to study how hardware issues affect these algorithms. Things like small errors in sensor positions, mismatches in gain or phase, or limited digital precision can reduce performance. Understanding and correcting for these real-world imperfections will help make the algorithms more robust.
By exploring these directions, future research can create DOA estimation methods that are not only accurate and fast but also flexible enough to handle real-world challenges combining the best features of both MUSIC and ESPRIT

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