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# Shape Transitions And Jacobian Instability In Excited Medium Mass Nuclei

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The study examines the Jacobi shape transition from a noncollective oblate form to a super or hyperdeformed collective prolate or triaxial shape occurring in rotating nuclei, similar to the behavior observed in gravitating rotating stars, specifically in Strontium isotopes such as ${}_{38}Sr^{80}$, ${}_{38}Sr^{82}$, ${}_{38}Sr^{84}$, ${}_{38}Sr^{86}$, ${}_{38}Sr^{88}$. To identify this transition, the cranked Nilsson-Strutinsky method is employed. In these calculations, the technique of adjusting the angular velocity to achieve a fixed spin is applied. Pairing effects are disregarded, as the Jacobi transition takes place only at very high spin levels where pairing correlations are no longer present. Our findings indicate that the isotopes analyzed in this study are promising candidates for observing the Jacobi shape transition.

## 1. Introduction

The investigation into structural alterations of nuclei under conditions of high excitation energy and significant angular momentum has ushered us into a new era in the field of nuclear structure physics. The experimental examination of the giant dipole resonance associated with excited states has begun to provide insights into the shape transitions occurring within these nuclei. The interplay of spin and temperature has resulted in a diverse array of shape transition phenomena in nuclei. Recently, one notable shape transition from a noncollective oblate configuration to a highly deformed collective prolate or nearly prolate (triaxial) shape has been both predicted and observed. This transition, akin to the Jacobi transition observed in gravitating rotating stars, has sparked considerable interest in recent times. The prediction of such a Jacobi transition in $^{45}Sc$ by Alhassid [1], along with its subsequent experimental validation by the Seattle group [2], has further ignited our curiosity in exploring these intriguing shape transitions within medium mass nuclei.

The aim of this work is to detect the possibility of the so called Jacobi transition in even–even medium mass nuclei namely Strontium isotopes ${}_{38}Sr^{80}$, ${}_{38}Sr^{82}$, ${}_{38}Sr^{84}$, ${}_{38}Sr^{86}$, ${}_{38}Sr^{88}$. For this purpose, we use the cranked Nilsson-Strutinsky method modified suitably to take in large deformations. In order to fix the spin in our calculations we use the method [3,4] of tuning the angular velocity. The rotating liquid drop model (RLDM) [5,6] has already predicted that nuclei should experience a shape transition at very high spins from noncollective oblate to collective prolate (or nearly prolate) with the superdeformed major to minor axes ratio of 2:1 or more. The shape evolution of rotating nuclei ultimately produces the above shape transition called the Jacobi shape transition. The Jacobi transition is not only a shape transition but it is a second order phase transition from noncollective to collective phases in nuclei. It is further interesting to note that such a shape transition is analogous to the Jacobi shape instability occurring in gravitating rotating stars [7]. Thus this study is very interesting and most important.

The second section of this theoretical study gives the theoretical framework for obtaining potential energy surfaces of the considered nuclei as a function of deformation (β) and nonaxiality (γ) parameters at different spins by the Strutinsky method. The third section gives a short description of the Jacobi transition and its detection. In the last section the results obtained for even-even medium mass strontium isotopes are presented and discussed in relation to the occurrence of Jacobi shape transition in these nuclei.

## 2. The Method

The shell energy calculations [8] for the nonrotating case (I = 0) assumes a single particle field

$$H_0 = \sum h_0 \quad (1)$$

where $h_0$ is the triaxial Nilsson Hamiltonian given by [9]

$$h_0 = \frac{P^2}{2m} + \frac{1}{2} m \sum_{i=1}^{3} \omega_i^2 x_i^2 + Cls + D(l^2 - 2\langle l^2 \rangle) \quad (2)$$

The three oscillator frequencies $\omega_i$ are given by Hill Wheeler parameterization as

$$\omega_x = \omega_0 \exp\left(-\sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma - \frac{2}{3}\pi\right)\right),$$
$$\omega_y = \omega_0 \exp\left(-\sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma - \frac{4}{3}\pi\right)\right),$$
$$\omega_z = \omega_0 \exp\left(-\sqrt{\frac{5}{4\pi}}\beta\cos\gamma\right)$$

with the constraint of constant volume for equipotentials

$$\omega_x \omega_y \omega_z = \mathring{\omega}_0^3 = \text{constant} \quad (3)$$

The following values [10] are chosen for the Nilsson parameters , μ and $\hbar\mathring{\omega}_0$:

κ = 0.093 and μ =0.15,

$$\hbar\overset{\circ}{\omega}_0 = \frac{45.3\ \text{MeV}}{(A^{1/3} + 0.77)} \quad (4)$$

The same values are used for protons as well as neutrons. It may be noted that in $h_0$ [Eq.(2)] the factor in front of $\langle l^2 \rangle$ term has been doubled to obtain better agreement between the Strutinsky-smoothed moment of inertia and the rigid rotor value (here within 10%). Accordingly the parameter D has been redetermined with the help of single-particle levels in the mass region indicated. The Hamiltonian in equation (2) is diagonalized in cylindrical representation [11,12] up to N =11 shells.

For the rotating case ( $I \neq 0$) the Hamiltonian becomes

$$H_\omega = H_0 - \omega J_z = \sum h_\omega, \quad (5)$$

Where

$$h_\omega = h_0 - \omega j_z \quad (6)$$

if it is assumed that the rotation takes place around the Z axis.

The single particle energy $e_i^\omega$ and wave function $\emptyset_i^\omega$ are given by

$$h_\omega \emptyset_i^\omega = e_i^\omega \emptyset_i^\omega \quad (7)$$

The spin projections are obtained as

$$\langle m_i \rangle = \langle \emptyset_i^\omega | j_z | \emptyset_i^\omega \rangle \quad (8)$$

The total shell energy is given by

$$E_{sp} = \sum \langle \emptyset_i^\omega | h_0 | \emptyset_i^\omega \rangle = \sum \langle e_i \rangle \quad (9)$$

Where

$$e_i^\omega = \langle e_i \rangle - \hbar\, \omega \langle m_i \rangle \quad (10)$$

Thus

$$E_{sp} = \sum e_i^\omega + \hbar\omega I \quad (11)$$

The total spin I is given by

$$I = \sum \langle m_i \rangle. \quad (12)$$

Since the difficulties encountered in the evaluation of total energy for large deformations through the summation of single particle energies for I = 0 case may be present for $I \neq 0$ case also [4,13], we use the Strutinsky shell correction method adopted to $I \neq 0$ case by suitably tuning the angular velocities to yield fixed spins [14]. For unsmoothened single particle level distribution we have

$$I = \int_{-\infty}^{\lambda} g_2 de^{\omega} = \sum_i \langle m_i \rangle \qquad (13)$$

and

$$E_{sp} = \int_{-\infty}^{\lambda} g_1 e^{\omega} de^{\omega} + \hbar\omega I = \sum_i e_i^{\omega} + \hbar\omega I \qquad (14)$$

For the Strutinsky smeared single particle level distribution, Eqs. (13) and (14) transform into

$$\tilde{I} = \int_{-\infty}^{\lambda} \widetilde{g_2}\, de^{\omega} = \sum_i \langle \widetilde{m_i} \rangle \qquad (15)$$

and

$$\widetilde{E_{sp}} = \int_{-\infty}^{\lambda} \widetilde{g_1}\, e^{\omega} de^{\omega} + \hbar\omega\tilde{I} \qquad (16)$$

$$= \sum_i^N \widetilde{e_i^{\omega}} + \hbar\omega\tilde{I} \qquad (17)$$

In the tuning method we have adapted [3,4], the total spin is calculated as

$$I = \widetilde{I_z} = \sum_{\nu=1}^{N} \langle \widetilde{J_z} \rangle_{\nu}^{\omega} + \sum_{\pi=1}^{Z} \langle \widetilde{J_z} \rangle_{\pi}^{\omega} \qquad (18)$$

The above relation allows us to select numerically the $\omega$ values that correspond to the chosen integer or half integer spins. Obviously the corresponding frequency values $\omega(I)$ change from one deformation point to another and the corresponding calculation must be repeated accordingly.

The total energy is now given by

$$E_T = E_{RLDM} + \left(E_{sp} - \widetilde{E_{sp}}\right) \qquad (19)$$

where the rotating liquid drop energy at constant spin

$$E_{RLDM} = E_{LDM} - \tfrac{1}{2} I_{rig}\omega^2 + \hbar\omega\tilde{I} \qquad (20)$$

Here the liquid drop energy $E_{LDM}$ is given by the sum of Coulomb and surface energies as

$$E_{LDM}(\beta,\gamma) = [(B_s - 1) + 2\chi(B_c - 1)a_s]A^{2/3} \qquad (21)$$

where $B_s$ and $B_c$ are the relative surface and Coulomb energies of the nucleus. Both $B_s$ and $B_c$ are elliptic integrals which depend on the semi axes lengths. The values used for the parameters $a_s$ and $\chi$ are as follows: $a_s$=19.7 MeV and the fissility parameter $\chi = (Z^2/A)/45$ where Z and A are the charge and mass numbers of the nucleus; $I_{rig}$ is the rigid body moment of inertia defined by β and $\gamma$ including the surface diffuseness correction can be calculated as follows.

In the case of an ellipsoidal shape described by the deformation parameter β and the shape parameter $\gamma$, the semiaxes $R_x$ , $R_y$ , $R_z$ are given by

$$R_x = R_0 exp\left[\sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma - \frac{2\pi}{3}\right)\right]$$

$$R_y = R_0 exp\left[\sqrt{\frac{5}{4\pi}}\beta\cos\left(\gamma - \frac{4\pi}{3}\right)\right]$$

and

$$R_z = R_0 \exp\left[\sqrt{\frac{5}{4\pi}}\beta\cos\gamma\right] \qquad (22)$$

By volume conservation we have

$$R_x R_y R_z = {R_0^0}^3 \qquad (23)$$

where $R_0^0$ is the radius of the spherical nucleus.

The moment of inertia about the z axis is given by

$$\frac{I_{rig}(\beta,\gamma)+2Mb^2}{\hbar^2} = \frac{1}{5}\frac{AM(R_x^2+R_y^2)}{\hbar} + \frac{2Mb^2}{\hbar^2} \qquad (24)$$

where the diffuseness correction to the moment of inertia is 2 $Mb^2$ and the diffuseness parameter b = 0.87 fm.

Here

$$R_0^0 = r_0 A^{1/3} \qquad (r_0 = 1.16\text{fm}). \qquad (25)$$

The calculations are carried out by varying ω values in steps of $0.03\omega_0$ from ω = 0 to ω = $0.45\omega_0$ , $\omega_0$ being the oscillator frequency for tuning to fixed spins. Since we are interested mainly in Jacobi shape transition $\gamma$ is varied from -180° to -120° in steps of -10°, $\gamma$ = -180° corresponding to noncollective oblate and $\gamma$ = -120° corresponding to collective prolate. Since the Jacobi transition involves large deformation, β values are varied from β = 0.0 to β =1.2 in steps of 0.1.

## 3. Results and Discussion

In this work the constant spin potential energy surfaces were extracted for the even-even strontium isotopes, ${}_{38}Sr^{80}$, ${}_{38}Sr^{82}$, ${}_{38}Sr^{84}$, ${}_{38}Sr^{86}$, ${}_{38}Sr^{88}$, to study the shape evolution and also to look for the possible Jacobi shape transitions in them. An important problem in nuclear structure physics is the studies of the shape evolution taking place at extreme values of spin very close to the limit imposed by the fission process. Such problems can be studied by the γ-decay of the Giant Dipole Resonance (GDR) which, because of the coupling of quadrupole degrees of freedom of the nucleus, constitutes a basic probe for the effective nuclear shape. It is observed that heavy nuclei change their equilibrium shape from spherical (or prolate) to oblate, the size of the oblate deformation increases with angular momentum and at certain value the nucleus undergoes the fission process. In light and medium mass nuclei, besides this transition, it is expected to exhibit more exotic behaviour – the Jacobi shape transition. A sudden shape transition occurring from noncollective oblate to large collective prolate or triaxial at certain critical value of angular momentum leading to super or hyper deformations is the so called Jacobi shape transition [1]. We have chosen five even-even strontium isotopes in the nuclear mass region A=80-88, which are of recent interest because of the existence of various high spin phenomena in the nuclei falling in this region.

In this work, the shape evolutions in ${}_{38}Sr^{80}$, ${}_{38}Sr^{82}$, ${}_{38}Sr^{84}$, ${}_{38}Sr^{86}$, ${}_{38}Sr^{88}$ isotopes are studied as a function of spin using cranked Nilsson Strutinsky method. The zero temperature potential energy surfaces for these isotopes have been obtained by tuned spin Strutinsky procedure. In the calculations performed here the spin is varied from I= 0 to 60 ħ in steps of 2 ħ. The equilibrium deformations are displayed in the (□-□) plane. We use the Hill-Wheeler convention; $\gamma=-180^0$ to an oblate shape that rotates around the symmetry axis (noncollective) and $\gamma=-120^0$ to a prolate shape that rotates around an axis perpendicular to the symmetry axis (collective). Figures 1-5 show the equilibrium shape evolution of ${}_{38}Sr^{80}$, ${}_{38}Sr^{82}$, ${}_{38}Sr^{84}$, ${}_{38}Sr^{86}$ and ${}_{38}Sr^{88}$ isotopes respectively at different spins performed with the tuned spin cranked Nilsson Strutinsky method. We see from Fig.1 that the $^{80}Sr$ has a spherical shape at its ground state (I=0 ħ) with deformation β=0.0. As spin increases a shape transition take place to noncollective oblate at I=10 ħ with deformation β=0.1 and an elongation of the same shape occurs with increased deformation on further increase of spin upto I=30 ħ. As spin increases a shape transition occur from oblate to triaxial at I=40 ħ. Then a shape transition occur at I=60 ħ from the triaxial configuration to collective prolate ($\gamma=-120^0$) shape with β=0.6. A sharp Jacobi shape transition is not obtained in the case of $^{80}Sr$ but which happens via triaxial.

It is noted from figure 2 that, $^{82}Sr$ is oblate in its ground state upto I=10 ħ and the shape changes to oblate as spin increases to 20 ħ. As angular momentum increases, the oblate deformation also increases and acquires β=0.2 at I=40 ħ and which persists in the same configuration upto I=50 ħ. The Jacobi shape transition takes place from noncollective oblate to collective prolate with β=0.6 at an angular momentum I=60 ħ in $^{82}Sr$. As angular momentum increases, the $^{84}Sr$ nucleus (Fig.3) which is originally spherical at I=0, acquires oblate deformation, corresponding to an elongation upto β=0.3 for I=50 ħ. Beyond 50 ħ, the Jacobi shape transition takes place; the nucleus becomes prolate with β=0.6 at an angular momentum of 60 ħ. If we look at Fig.4, $^{86}Sr$ which is spherical at I=0 ħ and changes its shape to oblate at I=10 ħ. As spin increases, the deformation also increases and which attains β=0.3 at I=30 ħ. The Jacobi shape transition occurs at a critical spin of 50 ħ in this case with β changing from 0.3 (oblate noncollective) to 0.7 (prolate collective) leading to β=0.8 at I=60 ħ. It is noted from Fig.5 that, $^{88}Sr$ is spherical in its ground state and moderate excitations, becoming oblate and undergoes a transition to prolate at a spin of 50 ħ with β=0.8. It stays in the prolate shape with increased deformation as a function of spin. Jacobi shape transition takes place in this case at a critical spin of 50 ħ, which is leading to hyperdeformation is clearly seen.

## 4. Conclusions

We have performed a systematic study of shape transitions in the strontium isotopes within the framework of cranked Nilsson Strutinsky method. Jacobian instability is predicted in even-even strontium isotopes which occur within the spin range 50 and 60 ħ in all the considered isotopes. Lower mass isotope of strontium such as $^{80}$Sr is showing oblate – prolate transition via triaxial and sharp Jacobi shape transition is not obtained in this case. It can be concluded that, the study of the Jacobi shape transitions in medium mass nuclei will throw a lot of light on the phase transitions leading to super and hyper deformations occurring in them. We have identified the strontium region as a very fertile region to detect Jacobi shape transition. It could be interesting to see experimentally whether such phase transitions and large deformations in these nuclei can be detected through giant dipole resonance cross sections built on excited states.

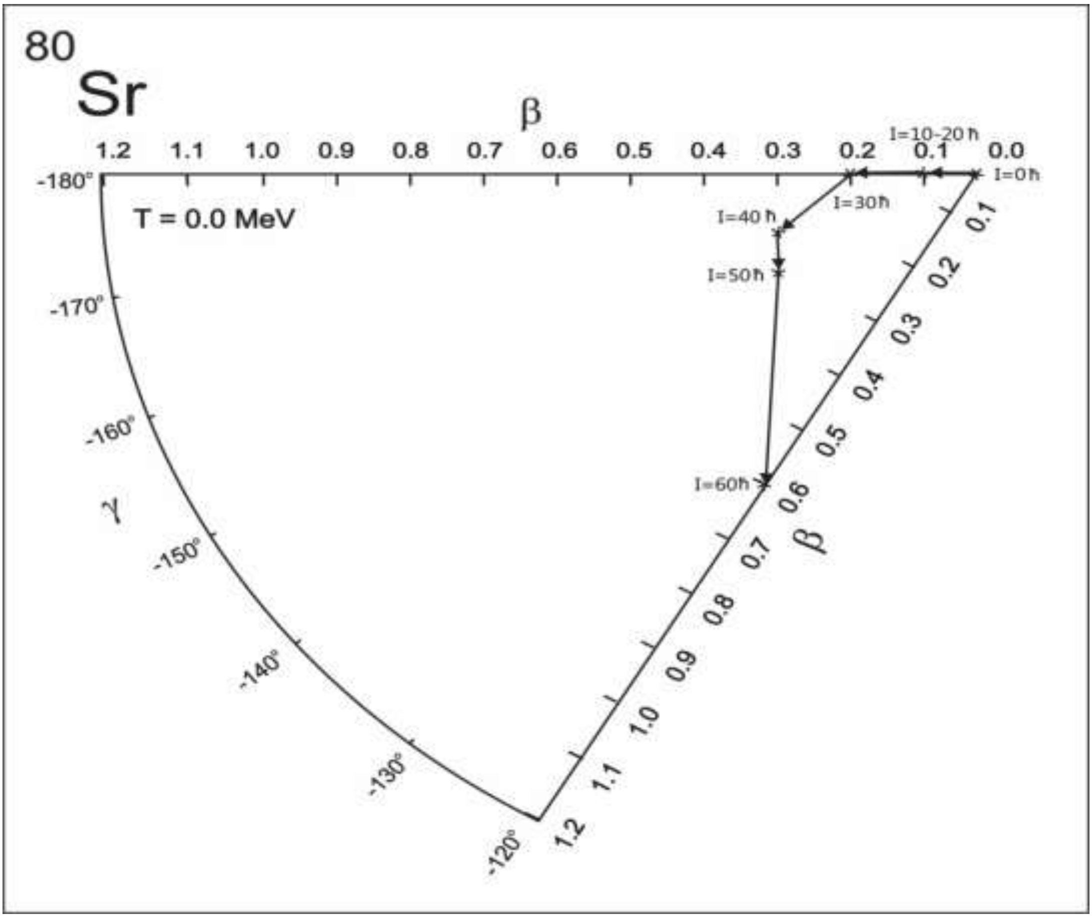


Fig. 1. Shape Transition and Jacobian Instability in highly excited $^{80}$Sr.

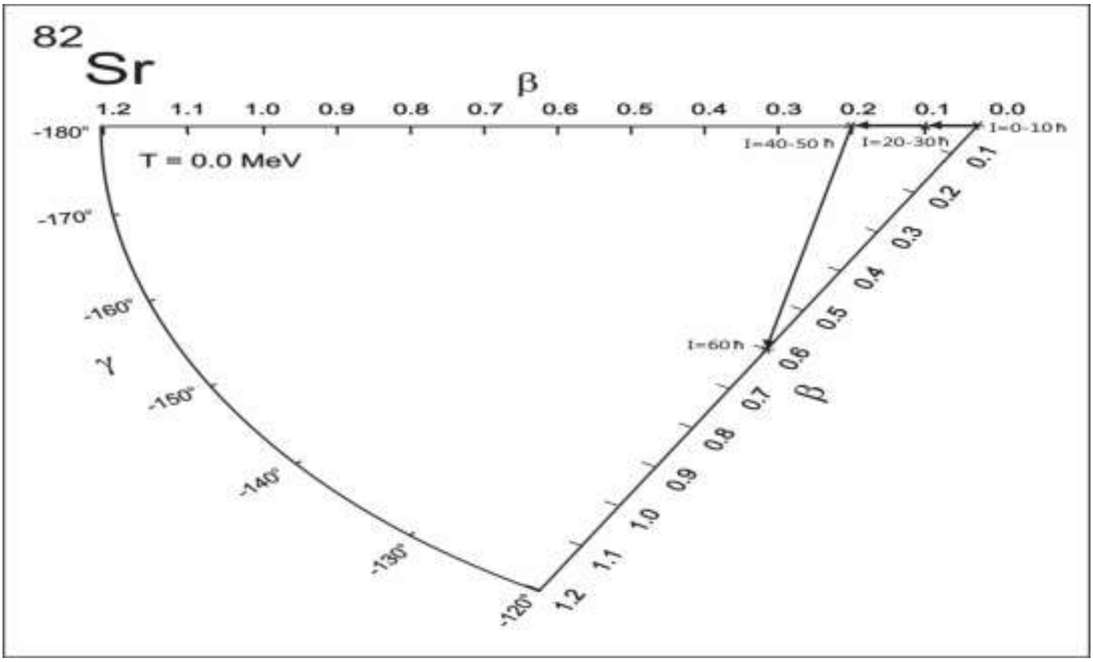


Fig. 2. Shape Transition and Jacobian Instability in highly excited $^{82}$Sr.

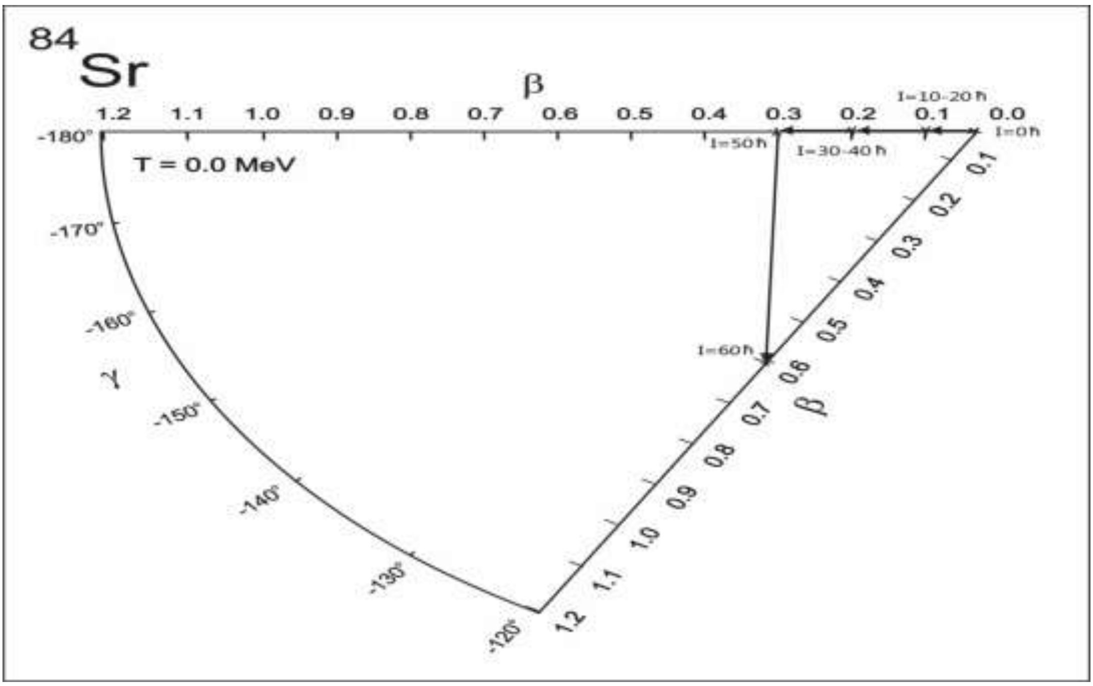


Fig. 3. Shape Transition and Jacobian Instability in highly excited $^{84}$Sr.

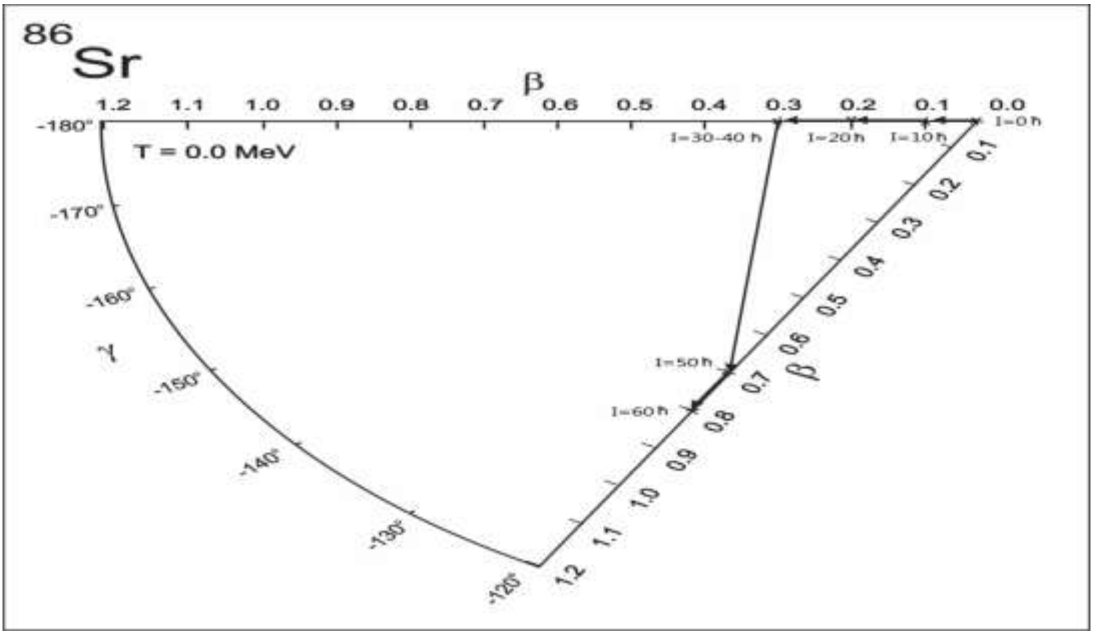


Fig. 4. Shape Transition and Jacobian Instability in highly excited $^{86}$Sr.

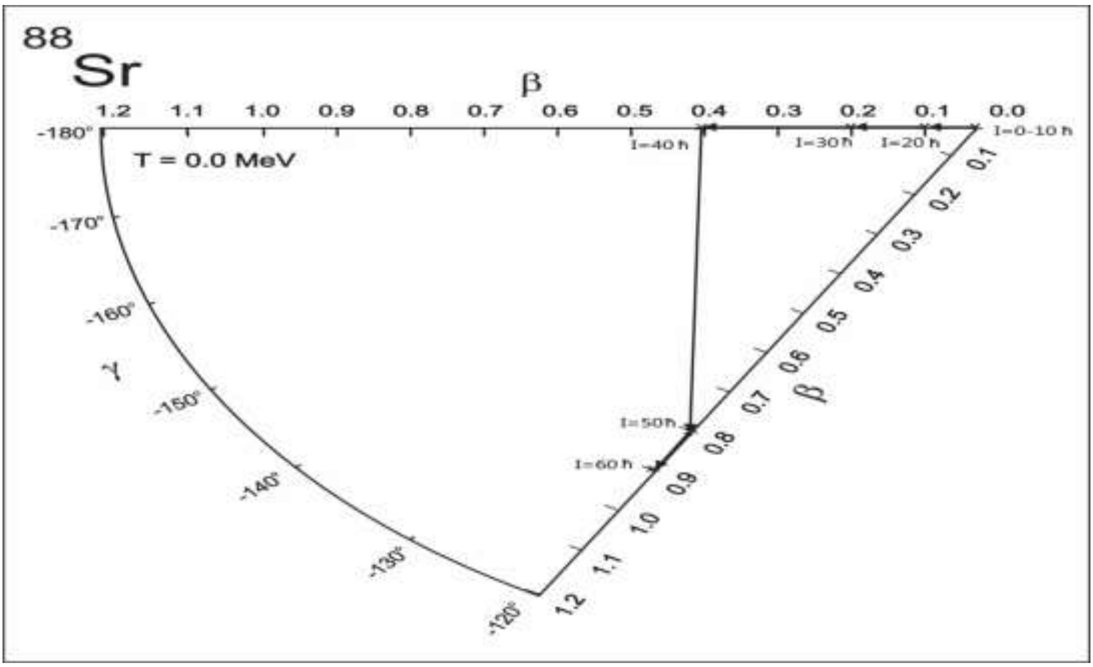


Fig. 5. Shape Transition and Jacobian Instability in highly excited $^{88}$Sr.

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