

An Exhaustive Ideational Exploration Of Dual Alpha Particle Emission

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Double alpha decay is an infrequent nuclear event in which an unstable atomic nucleus simultaneously emits two alpha particles of same kind, causing a notable alteration in the atomic and mass numbers of the parent nuclei. First experimental limit on the double alpha decay of ²⁰⁹Bi isotope was reported in the year 2021, as $T_{1/2} > 2.9 \times 10^{20}$ years at 90% confidence level. Recently some of the articles provided the half-life($T_{1/2}$) limit of few other isotopes in 2024. Our CYE Model has been previously utilized to study alpha(α) decay, Cluster decay, Spontaneous Fission(SF), Super Heavy Elements(SHE) and also two proton(2p) decay, recently. Again we have also used our CYE Model to study double alpha(2α) decay recently from 2022. In this work, we have explored the double alpha decay properties of radioactive nuclei with atomic number range from $Z=60$ to $Z=93$ using Cubic plus Yukawa plus Exponential Model(CYEM) with incorporating deformation effects for both parent and daughter nuclei considering the emitted fragment spherical. Multiple theoretical frameworks (with and without deformation effects) have been proposed to scrutinize the occurrence of double alpha decay from several nuclei from 1979 to till date. Our calculated values are compared with other available theoretical models, and are in good agreement with each other. Furthermore, we have studied the half-life comparison of double alpha decay and ⁸Be decay from ²²⁹Pu, ²³⁰Am, ²³²Cm, ²³⁴Bk, ²³⁶Cf, ²³⁹Es, ²⁴¹Fm, ²⁴⁵Md isotopes using our CYE Model.

Keywords: Q-value, Half-life, Double alpha Decay, Cluster decay, Deformation effects.

1. INTRODUCTION:

The structure of an atomic nucleus can be discerned through the analysis of the radiations it emits. Radioactivity is an emerging field in the domain of Nuclear Physics to apprehend the stability of an atomic nucleus and it was initially discovered by Henri Becquerel in 1896[1], and it refers to the spontaneous emission of particles and electromagnetic radiation from the nucleus of an unstable atom. An atomic nucleus spontaneously emits a single particle as well as double particles during radioactive decay. In a single alpha decay, the unstable atomic nucleus undergoes a transformation into a more stable configuration by the emission of an

alpha particle which is a helium nucleus. In 1928, Gamow [2] formulated the theory of alpha decay, and Gurney and Condon independently formulated the alpha decay theory using the basis of quantum tunneling. Concurrent liberation of dual particles from an atomic nucleus is denominated as double particle decay. Analogous to 2β , 2γ , $2p$, $2n$ decays, the double alpha decay would fascinate many nuclear physicists theoretically[3-22] and experimentally[23-26]. Double alpha decay is a captivating phenomenon in nuclear physics, where a nucleus emits two identical alpha particles simultaneously, offering unique insights into nuclear structure and stability. Initially double alpha decay was debated by Yu. Novikov in 1979[3]. V.I. Tretyak [6] has reported, for the first time, an experimental half-life limit of double alpha decay of ^{209}Bi as 2.9×10^{20} years, utilizing data obtained from an experiment conducted by de Marcillac[28]. Different experimental methods are also suggested to measure the energies of emitted alpha particles during the nuclear reaction[6]. Again numerous theoretical frameworks are advanced to investigate the phenomenon of double alpha decay from the year 1979 to still now[3-27]. Very recently[25], P. Belli et.al., reported the double alpha decay half-life limits of ^{189}Os and ^{192}Os as $T_{1/2} > 10^{20}$ years for the first time. Again the same author[26], reported, half-life limits for double alpha decay of ^{148}Nd isotope as $T_{1/2} = 2.1 \times 10^{20}$ years.

Our CYE Model has a Cubic Potential in the pre scission region connected by Coulomb plus Yukawa plus exponential potential in the post-scission region. In our previous articles[18-23] we have presented the 2α -decay half-life of various nuclei using our CYE Model in two sphere approximation. For the current research, our model has been further upgraded by incorporating deformation parameters to precise determination of double alpha decay half-lives for the parent isotopes with atomic number varying from $Z=60$ to $Z=93$. In this work we have considered the shape of parent and daughter nuclei as spheroid and keeping the emitted cluster as spherical and calculated the half-lives from the range $Z=60$ to 93. Additionally we have compared the 2α decay and ^8Be decay properties of ^{229}Pu , ^{230}Am , ^{232}Cm , ^{234}Bk , ^{236}Cf , ^{239}Es , ^{241}Fm , ^{245}Md isotopes using our CYE Model, and compared with MGLDM and Universal Decay Law.

2. METHODOLOGY

In this work, to study the properties of double alpha decay, we have used a recently developed realistic model[29] called as Cubic plus Yukawa plus Exponential Model(CYEM). Here, the zero-point vibration energy is explicitly added without violating the conservation of energy and the inertial mass coefficient dependent on the centre of mass distance.

2.1. HALF LIFE:

The half-life(in seconds) of the system is calculated using the following relation,

$$T = \frac{1.433 \times 10^{-21} (1 + \exp K)}{E_v} \quad (1)$$

where action integral K is

$$K = K_L + K_R$$

where,

$$K_L = \frac{2}{\hbar} \int_{r_a}^{r_t} \sqrt{2B_r(r)V(r)} dr \quad (2)$$

$$K_R = \frac{2}{\hbar} \int_{r_t}^{r_b} \sqrt{2B_r(r)V(r)} dr \quad (3)$$

where r_a and r_b are the two appropriate zeros of the integrand.

Here E_v is zero point vibration energy and is given by

$$E_v = \frac{\pi \hbar}{2} \left[\frac{\left[\frac{2Q}{\mu} \right]^{1/2}}{(C_1 + C_2)} \right] \quad (4)$$

C_1 and C_2 are the central radii of the fragments given by,

$$C_i = 1.18A_i^{1/3} - 0.48 \quad (i=1,2) \quad (5)$$

and $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is the reduced mass.

2.2. POTENTIAL FOR THE POST-SCISSION REGION:

In this present study parent and daughter nuclei are considered as spheroid and the emitted fragment as spherical. If the parent and daughter nucleus have a deformation say quadrupole deformation only, and the Q value of the reaction is taken as the origin, then the potential for the post-scission is given by,

$$V(r) = V_c(r) + V_n(r) - V_{df}(r) - Q; \quad r \geq r_t \quad (6)$$

$V_c(r)$ is the Coulomb potential between a spheroidal daughter and spherical emitted fragment, $V_n(r)$ is the nuclear interaction energy due to finite range effects Krappe et.al. [30], $V_{df}(r)$ is the change in the nuclear interaction energy due to quadrupole deformation of the daughter nucleus.

For a prolate spheroid daughter nucleus with longer axis along the fission direction, Pik-Pichak[31]

$$V_c(r) = \frac{3}{2} \frac{Z_1 Z_2 e^2}{r} \left[\frac{1-r^2}{2} \ln \frac{\gamma+1}{\gamma-1} + \gamma \right] \quad (7)$$

For an oblate spheroid daughter nucleus with shorter axis along the fission direction

$$V_c(r) = \frac{3}{2} \frac{Z_1 Z_2 e^2}{r} [\gamma(1 + \gamma^2) \arctan \gamma^{-1} - \gamma^2] \quad (8)$$

$$\text{where } \gamma = \frac{r}{(a_2^2 - b_2^2)^{\frac{1}{2}}}$$

where a_2 and b_2 are the semi-major and minor axes of the spheroidal daughter nucleus respectively.

If the nuclei have spheroid shape, the radius vector $R(\theta)$ making an angle θ with the axis of symmetry locating sharp surface of a deformed nuclei is given by ref[31]

$$R(\theta) = R_0 [1 + \sum_{n=0}^{\infty} \sum_{m=-n}^n \beta_{nm} Y_{nm}(\theta)] \quad (9)$$

where R_0 is the radius of the equivalent spherical nucleus.

2.3. POTENTIAL FOR THE PRE-SCISSION REGION:

The potential for the pre scission region which connects the ground state and contact point is approximated by a third order polynomial in r suggested by Nix[32] having the form,

$$V(r) = -E_v + V(r_t) + \left\{ s_1 \left[\frac{r-r_i}{r_t-r_i} \right]^2 - s_2 \left[\frac{r-r_i}{r_t-r_i} \right]^3 \right\} \quad (10)$$

Where r_i is the distance between the centre of mass of two portions of the daughter and the emitted nuclei in the spheroidal parent nucleus.

If we consider spheroid deformation β_2 , then

$$R(\theta) = R_0 \left[1 + \beta_2 \left[\frac{5}{4\pi} \right]^{\frac{1}{2}} \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) \right] \quad (11)$$

If the Nilsson's hexadecapole deformation β_4 is also included in the parent deformation, then the radius becomes,

$$R(\theta) = \left[\left[1 + \beta_2 \left[\frac{5}{4\pi} \right]^{\frac{1}{2}} \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) \right] + \left[1 + \beta_4 \left[\frac{9}{4\pi} \right]^{\frac{1}{2}} \frac{1}{8} (35 \cos^4 \theta - 30 \cos^2 \theta + 3) \right] \right] \quad (12)$$

Hence the half lives of nuclei are calculated using equation no.(1)

Table-1: Logarithmic half life values of our CYE Model with other available models including deformation.

Parent nuclei	Daughter nuclei	$Q_{2\alpha}$ (MeV)	Log $T_{1/2}$ (years)			
			CYEM [WOD] [23]	CYEM [WP&D] [present work]	CPPM [8]	MGLDM [8]
^{145}Nd	$^{137}_{56}\text{Ba}_{81}$	1.4395	143.01	142.63	142.32	139.73
^{146}Nd	$^{138}_{56}\text{Ba}_{82}$	2.4859	92.03	91.07	89.42	87.59
^{147}Sm	$^{139}_{58}\text{Ce}_{81}$	2.8317	84.27	83.34	83.50	81.23
^{148}Sm	$^{140}_{58}\text{Ce}_{82}$	3.8900	60.20	56.76	59.40	57.77
^{149}Sm	$^{141}_{58}\text{Ce}_{83}$	3.4473	68.87	66.510	68.10	66.18
^{150}Sm	$^{142}_{58}\text{Ce}_{84}$	2.6322	90.28	89.99	88.98	87.52
^{152}Sm	$^{144}_{58}\text{Ce}_{86}$	0.8194	224.39	222.23	221.01	220.24
^{151}Eu	$^{143}_{59}\text{Pr}_{84}$	3.5656	68.45	67.04	66.96	65.51
^{153}Eu	$^{145}_{59}\text{Pr}_{86}$	1.4089	156.06	153.41	153.21	150.39
^{152}Gd	$^{144}_{60}\text{Nd}_{84}$	4.1912	58.82	56.42	57.77	56.28
^{154}Gd	$^{146}_{60}\text{Nd}_{86}$	2.3701	104.71	103.03	100.96	98.92
^{155}Gd	$^{147}_{60}\text{Nd}_{87}$	1.2270	176.78	174.69	172.68	170.54
^{156}Dy	$^{148}_{62}\text{Sm}_{86}$	3.9575	66.76	65.41	62.75	60.77
^{158}Dy	$^{150}_{62}\text{Sm}_{88}$	1.7940	138.59	136.72	133.96	131.70
^{162}Er	$^{154}_{64}\text{Gd}_{90}$	2.5216	109.28	107.72	104.08	101.76
^{164}Er	$^{156}_{64}\text{Gd}_{92}$	1.7422	148.13	146.58	142.26	139.88
^{166}Er	$^{158}_{64}\text{Gd}_{94}$	0.9137	235.62	233.38	229.97	228.75
^{169}Tm	$^{161}_{66}\text{Tb}_{95}$	1.3365	184.40	182.92	179.12	176.32

^{168}Yb	$^{160}_{66}\text{Dy}_{94}$	3.2410	91.01	89.73	85.76	83.30
^{170}Yb	$^{162}_{66}\text{Dy}_{96}$	2.5677	112.51	111.27	107.38	104.69
^{171}Yb	$^{163}_{66}\text{Dy}_{97}$	2.2245	127.06	125.77	121.97	119.18
^{172}Yb	$^{164}_{66}\text{Dy}_{98}$	1.8627	147.52	145.33	141.91	138.89
^{173}Yb	$^{165}_{66}\text{Dy}_{99}$	1.2116	201.68	200.48	197.15	193.96
^{174}Yb	$^{166}_{66}\text{Dy}_{100}$	0.7904	269.20	267.73	265.46	262.05
^{175}Lu	$^{167}_{67}\text{Ho}_{100}$	2.2652	127.84	126.89	123.48	120.29
^{176}Lu	$^{168}_{67}\text{Ho}_{101}$	1.8290	151.74	150.82	147.75	144.34
^{174}Hf	$^{166}_{68}\text{Er}_{98}$	4.2317	72.90	72.011	68.63	65.84
^{176}Hf	$^{168}_{68}\text{Er}_{100}$	3.5650	86.94	86.20	82.83	79.76
^{177}Hf	$^{169}_{68}\text{Er}_{101}$	3.1927	96.64	95.88	92.86	89.60
^{178}Hf	$^{170}_{68}\text{Er}_{102}$	2.8236	108.10	107.39	104.40	100.99
^{179}Hf	$^{171}_{68}\text{Er}_{103}$	2.4062	124.13	123.49	120.80	117.17
^{180}Hf	$^{172}_{68}\text{Er}_{104}$	1.8544	153.14	152.52	150.27	146.35
^{180}Ta	$^{172}_{69}\text{Tm}_{103}$	3.5917	88.31	87.871	84.94	81.46
^{181}Ta	$^{173}_{69}\text{Tm}_{104}$	2.9679	105.66	105.141	102.90	99.14
^{180}W	$^{172}_{70}\text{Yb}_{102}$	4.7695	67.17	67.141	64.04	60.68
^{182}W	$^{174}_{70}\text{Yb}_{104}$	3.8486	84.41	84.03	81.64	77.96
^{183}W	$^{175}_{70}\text{Yb}_{105}$	3.4801	93.17	92.78	90.78	86.90
^{184}W	$^{176}_{70}\text{Yb}_{106}$	2.9361	109.01	108.77	107.13	102.91
^{186}W	$^{178}_{70}\text{Yb}_{108}$	2.3370	132.49	132.29	130.98	126.51
^{185}Re	$^{177}_{71}\text{Lu}_{106}$	3.7149	89.46	89.22	87.75	83.61
^{187}Re	$^{179}_{71}\text{Lu}_{108}$	2.9926	109.46	109.29	108.22	103.75
^{184}Os	$^{176}_{72}\text{Hf}_{104}$	5.4740	60.21	59.96	57.96	54.29

^{168}Yb	$^{160}_{66}\text{Dy}_{94}$	3.2410	91.01	89.73	85.76	83.30
^{186}Os	$^{178}_{72}\text{Hf}_{106}$	4.2855	79.24	79.14	77.58	73.41
^{187}Os	$^{179}_{72}\text{Hf}_{107}$	4.3941	77.15	77.03	75.69	71.52
^{188}Os	$^{180}_{72}\text{Hf}_{108}$	3.7923	89.64	89.56	88.62	84.17
^{189}Os	$^{181}_{72}\text{Hf}_{109}$	3.5663	95.11	95.03	94.11	89.61
^{190}Os	$^{182}_{72}\text{Hf}_{110}$	2.4919	130.78	130.68	130.26	125.37
^{191}Ir	$^{183}_{73}\text{Ta}_{110}$	3.7342	93.01	87.337	92.61	87.88
^{193}Ir	$^{185}_{73}\text{Ta}_{112}$	2.0080	158.49	152.26	158.62	153.40
^{190}Pt	$^{182}_{74}\text{W}_{108}$	6.0898	55.52	55.38	54.33	50.33
^{192}Pt	$^{184}_{74}\text{W}_{110}$	4.5671	77.58	75.54	77.30	72.71
^{194}Pt	$^{186}_{74}\text{W}_{112}$	2.8986	119.70	119.59	119.82	114.77
^{195}Pt	$^{187}_{74}\text{W}_{113}$	2.2633	146.97	140.57	147.42	142.23
^{196}Pt	$^{188}_{74}\text{W}_{114}$	1.1735	237.97	237.76	239.03	233.54
^{197}Au	$^{189}_{75}\text{Re}_{114}$	1.9895	165.55	160.28	166.61	160.87
^{196}Hg	$^{188}_{76}\text{Os}_{112}$	4.4615	83.19	78.60	83.69	78.46
^{198}Hg	$^{190}_{76}\text{Os}_{114}$	2.9036	124.27	119.36	125.11	119.51
^{199}Hg	$^{191}_{76}\text{Os}_{115}$	1.9993	167.88	163.01	168.95	163.18
^{200}Hg	$^{192}_{76}\text{Os}_{116}$	1.5291	206.60	205.59	205.79	199.92
^{201}Hg	$^{193}_{76}\text{Os}_{117}$	0.8819	296.97	291.46	281.16	281.16
^{203}Tl	$^{195}_{77}\text{Ir}_{118}$	1.0810	264.05	258.77	265.67	259.65
^{204}Pb	$^{196}_{78}\text{Pt}_{118}$	2.6848	137.70	133.04	138.85	133.13
^{206}Pb	$^{198}_{78}\text{Pt}_{120}$	1.2686	240.82	238.45	242.60	236.16

^{209}Bi	$^{201}_{79}\text{Au}_{122}$	3.2922	117.79	115.50	119.14	113.41
^{191}At	$^{183}_{81}\text{Tl}_{102}$	15.6012	15.50	8.018	-	4.8759
^{192}At	$^{184}_{81}\text{Tl}_{103}$	14.9592	17.52	10.03	-	6.7316
^{193}At	$^{185}_{81}\text{Tl}_{104}$	14.8412	17.87	10.3712	-	7.0682
^{194}At	$^{186}_{81}\text{Tl}_{105}$	14.3172	19.63	12.08	-	8.7613

^{195}At	$^{187}_{81}\text{Tl}_{106}$	14.1252	20.28	12.72	-	9.3809
^{194}Rn	$^{186}_{82}\text{Pb}_{104}$	15.5562	8.89	8.314	-	-
^{210}Rn	$^{202}_{82}\text{Pb}_{120}$	17.7472	1.96	1.801	-	-
^{211}Rn	$^{203}_{82}\text{Pb}_{121}$	15.7962	7.45	7.34	-	-
^{215}Rn	$^{207}_{82}\text{Pb}_{125}$	16.4332	5.39	4.368	-	-
^{216}Rn	$^{208}_{82}\text{Pb}_{126}$	17.1517	3.32	2.396	-	-
^{217}Rn	$^{209}_{82}\text{Pb}_{127}$	16.4238	5.34	4.040	-	-
^{218}Rn	$^{210}_{82}\text{Pb}_{128}$	15.096	9.44	8.4312	-	-
^{209}Fr	$^{201}_{83}\text{Bi}_{118}$	18.4372	1.02	0.7672	-	-
^{210}Fr	$^{202}_{83}\text{Bi}_{119}$	16.3442	6.67	6.1461	-	-
^{211}Fr	$^{203}_{83}\text{Bi}_{120}$	17.5072	3.35	2.623	-	-
^{215}Fr	$^{207}_{83}\text{Bi}_{124}$	15.5228	9.02	7.990	-	-
^{216}Fr	$^{208}_{83}\text{Bi}_{125}$	16.9914	4.58	3.675	-	-
^{217}Fr	$^{209}_{83}\text{Bi}_{126}$	17.7238	2.55	1.279	-	-
^{218}Fr	$^{210}_{83}\text{Bi}_{127}$	17.0011	4.48	3.087	-	-
^{219}Fr	$^{211}_{83}\text{Bi}_{128}$	15.6262	8.55	6.812	-	-
^{209}Ra	$^{201}_{84}\text{Po}_{117}$	20.5112	-4.99	-3.36	-	-
^{210}Ra	$^{202}_{84}\text{Po}_{118}$	21.8522	-6.20	-6.39	-	-
^{211}Ra	$^{203}_{84}\text{Po}_{119}$	19.6012	-2.93	-2.83	-	-
^{216}Ra	$^{208}_{84}\text{Po}_{124}$	15.9104	8.62	7.344	-	-
^{217}Ra	$^{209}_{84}\text{Po}_{125}$	17.4062	4.20	3.74	-	-
^{218}Ra	$^{210}_{84}\text{Po}_{126}$	17.7493	3.23	2.74	-	-
^{219}Ra	$^{211}_{84}\text{Po}_{127}$	16.9767	5.30	3.835	-	-
^{220}Ra	$^{212}_{84}\text{Po}_{128}$	15.7916	8.84	7.791	-	-
^{209}Ac	$^{201}_{85}\text{At}_{116}$	22.3392	-7.99	-6.22	-	-
^{210}Ac	$^{202}_{85}\text{At}_{117}$	19.8052	-1.24	-1.77	-	-
^{211}Ac	$^{203}_{85}\text{At}_{118}$	21.1932	-4.19	-4.34	-	-
^{217}Ac	$^{209}_{85}\text{At}_{124}$	16.7362	6.91	6.90	-	-
^{218}Ac	$^{210}_{85}\text{At}_{125}$	17.9722	3.43	2.78	-	-

^{219}Ac	$^{211}_{85}\text{At}_{126}$	18.3674	2.38	1.65	-	-
^{219}Ac	$^{211}_{85}\text{At}_{126}$	18.4	2.29	0.919	-	-
^{220}Ac	$^{212}_{85}\text{At}_{127}$	17.5223	4.57	3.72	-	-
^{220}Ac	$^{212}_{85}\text{At}_{127}$	17.5	4.63	3.78	-	-
^{211}Th	$^{203}_{86}\text{Rn}_{117}$	23.3842	-8.64	-7.23	-	-
^{218}Th	$^{210}_{86}\text{Rn}_{124}$	17.1222	6.58	6.49	-	-
^{219}Th	$^{211}_{86}\text{Rn}_{125}$	18.3652	3.15	2.66	-	-
^{220}Th	$^{212}_{86}\text{Rn}_{126}$	18.4992	2.77	2.00	-	-
^{220}Th	$^{212}_{86}\text{Rn}_{126}$	18.5	2.77	1.99	-	-
^{221}Th	$^{213}_{86}\text{Rn}_{127}$	17.7862	4.61	2.976	-	-
^{221}Th	$^{213}_{86}\text{Rn}_{127}$	17.8	4.57	2.939	-	-
^{232}Th	$^{224}_{86}\text{Rn}_{138}$	8.1519	50.46	48.181	45.62	42.59

^{218}Pa	$^{210}_{87}\text{Fr}_{123}$	17.1442	7.32	6.682	-	-
^{219}Pa	$^{211}_{87}\text{Fr}_{124}$	17.8702	5.25	4.8312	-	-
^{220}Pa	$^{212}_{87}\text{Fr}_{125}$	18.9442	2.41	1.581	-	-
^{221}Pa	$^{213}_{87}\text{Fr}_{126}$	19.0742	2.05	1.3010	-	-
^{222}Pa	$^{214}_{87}\text{Fr}_{127}$	18.1682	4.33	2.695	-	-
^{222}Pa	$^{214}_{87}\text{Fr}_{127}$	18.1	4.52	2.865	-	-
^{223}Pa	$^{215}_{87}\text{Fr}_{128}$	17.1722	7.06	5.951	-	-
^{231}Pa	$^{223}_{87}\text{Fr}_{136}$	10.1922	36.46	34.28	31.68	29.01
^{219}U	$^{211}_{88}\text{Ra}_{123}$	17.6142	6.76	5.827	-	-
^{234}U	$^{226}_{88}\text{Ra}_{138}$	9.6274	41.39	33.02	35.72	32.86
^{235}U	$^{227}_{88}\text{Ra}_{139}$	8.8913	46.84	44.42	41.03	38.00
^{238}U	$^{230}_{88}\text{Ra}_{142}$	7.9416	54.84	52.35	48.73	45.53
^{220}Np	$^{212}_{89}\text{Ac}_{123}$	18.3302	5.55	4.652	-	-
^{221}Np	$^{213}_{89}\text{Ac}_{124}$	18.9192	3.97	2.901	-	-
^{222}Np	$^{214}_{89}\text{Ac}_{125}$	19.9872	1.31	0.55199	-	-
^{223}Np	$^{215}_{89}\text{Ac}_{126}$	19.7792	1.76	1.0711	-	-
^{224}Np	$^{216}_{89}\text{Ac}_{127}$	19.0322	3.56	2.066	-	-
^{225}Np	$^{217}_{89}\text{Ac}_{128}$	18.0682	6.06	4.841	-	-
^{226}Np	$^{218}_{89}\text{Ac}_{129}$	17.1202	8.73	7.5211	-	-

Figures 1-14 : Plot connecting the decay energy and Log $T_{1/2}$ of several nuclei using our CYE Model and is compared with other available models.

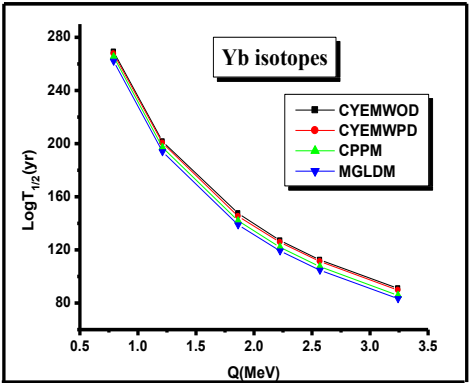


Fig.1

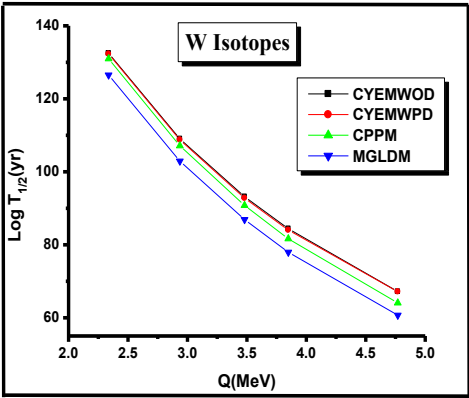


Fig.2

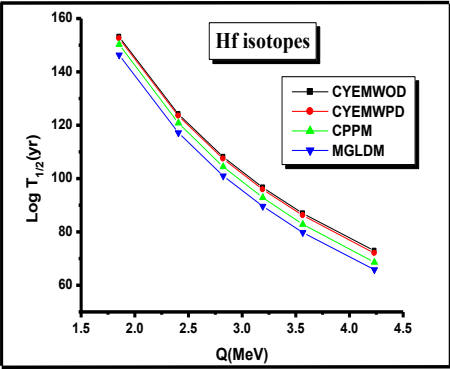
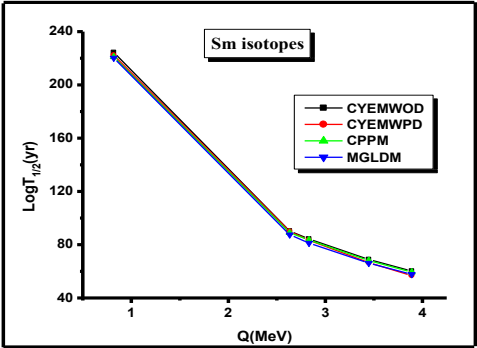


Fig.4



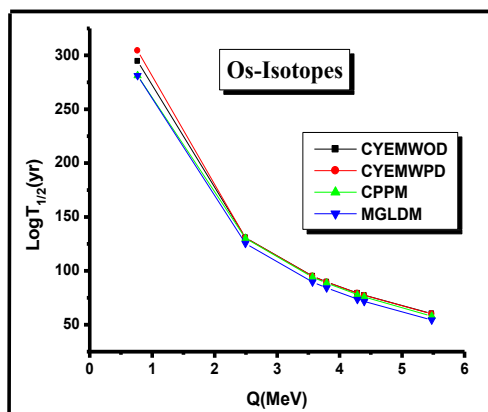


Fig.5

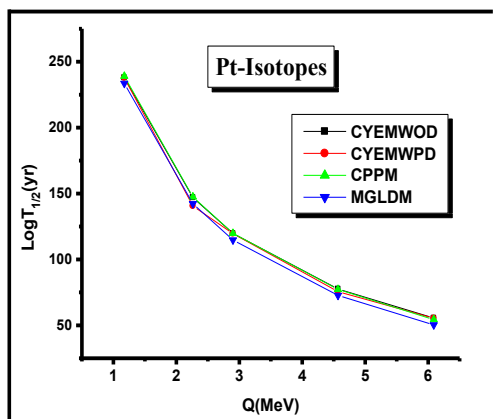


Fig.6

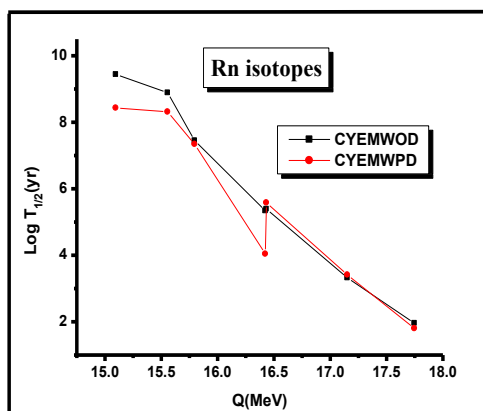


Fig.7

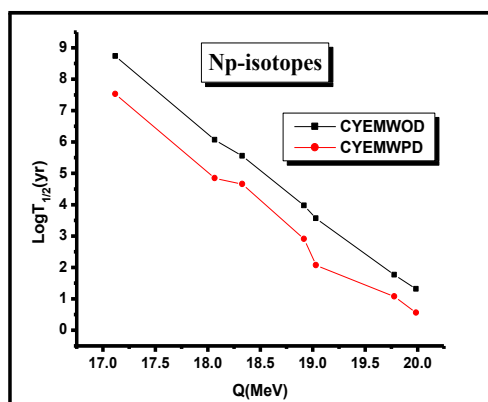


Fig.8

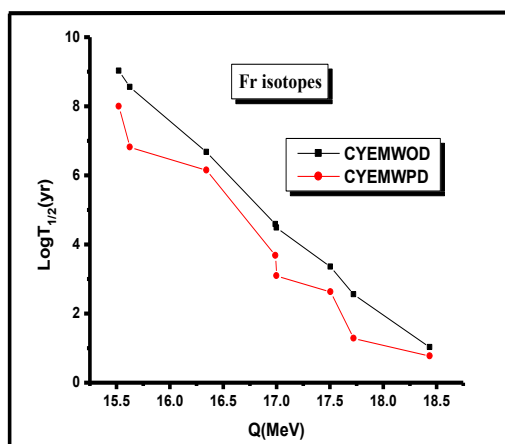


Fig.9

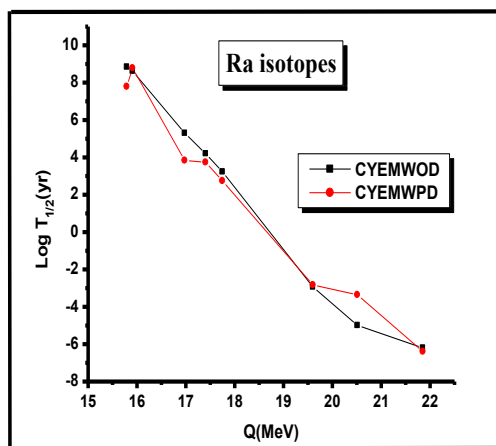


Fig.10

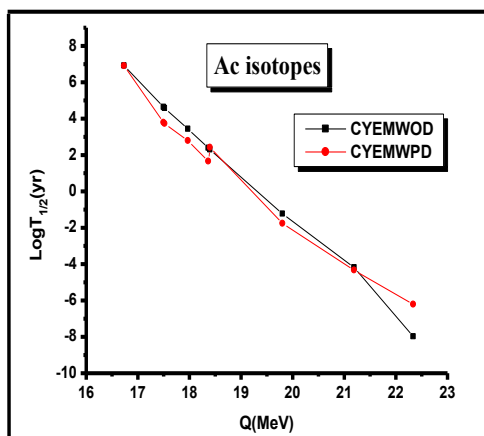


Fig.11

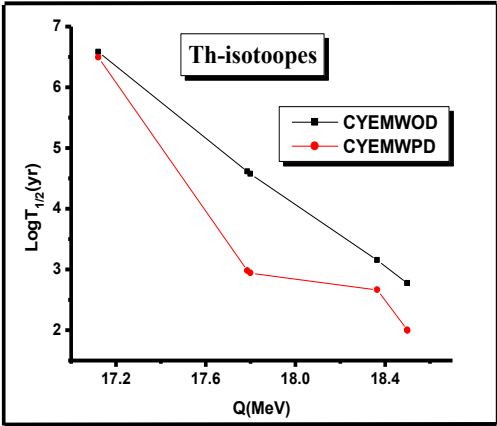


Fig.12

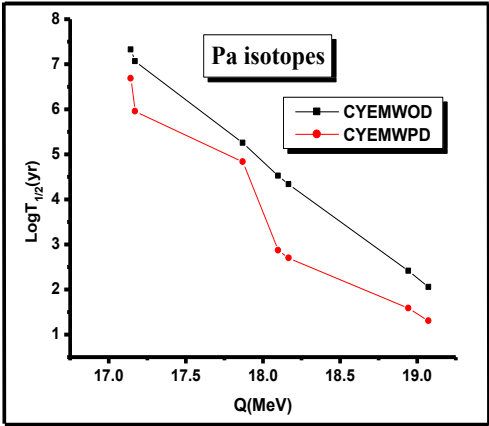


Fig.13

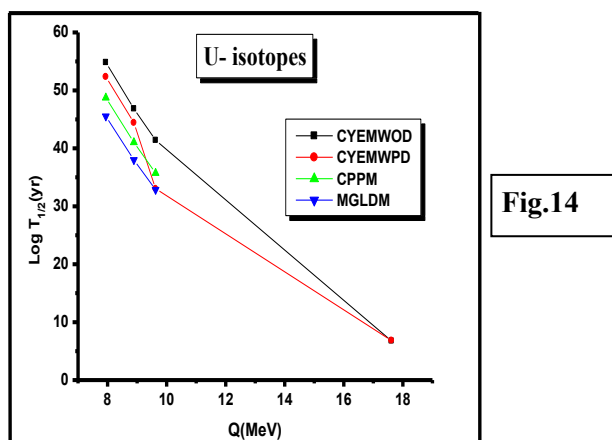


Table-2: Half life comparison of 2α and ^8Be decay from various nuclei.

Parent Nuclei	Daughter nuclei	Emitted nuclei	Q _{2α} (MeV)	Log T _{1/2} (s)		
				CYEM Calc.	MGLDM [14]	UDL [14]
²²⁹ Pu	²²¹ Th	⁸ Be	15.4840	22.24	19.073	21.398
		2 α	15.5759	21.92	18.765	21.102
²³⁰ Am	²²² Pa	⁸ Be	16.0125	21.26	18.091	20.491
		2 α	16.1044	20.95	17.796	20.206
²³² Cm	²²⁴ U	⁸ Be	16.0247	22.00	18.768	21.189
		2 α	16.1166	21.69	18.469	20.901
²³⁴ Bk	²²⁶ Np	⁸ Be	16.1360	22.41	19.120	21.274
		2 α	16.2279	22.10	21.286	21.286
²³⁶ Cf	²²⁸ Pu	⁸ Be	16.5405	21.83	18.526	21.045
		2 α	16.6324	21.53	18.235	20.764
²³⁹ Es	²³¹ Am	⁸ Be	16.7381	21.91	18.563	21.133
		2 α	16.8299	21.61	18.275	20.854
²⁴¹ Fm	²³³ Cm	⁸ Be	17.1980	21.14	17.826	20.464
		2 α	17.2899	20.85	17.547	20.193
²⁴⁵ Md	²³⁷ Bk	⁸ Be	17.4086	21.18	17.794	20.489
		2 α	17.5005	20.89	17.517	20.221

Figure 15. Plot for double alpha decay and beryllium decay comparison of various isotopes using CYEM.

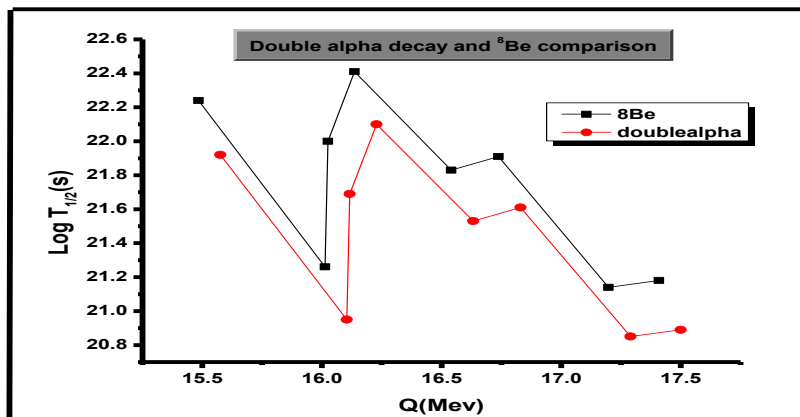


Fig.15

3. RESULTS AND DISCUSSIONS:

A radioactive nucleus will attempt to reach stability by ejecting nucleons (protons, neutrons) as well as other particles, or by releasing energy in the form of radiation is called radioactivity. Double alpha decay is a significant mechanism in nuclear transition and is studied for several nuclei using our CYE model[33] in two sphere approximation and also including of deformation effects. Decay energy($Q_{2\alpha}$) is an important factor to determine the half-life of an atomic nuclei during nuclear decay. Table-1 shows the half life time comparison of our CYE Model in two sphere approximation, including deformation effects for both parent and daughter nuclei with Coulomb Proximity Potential Model and Modified Generalized Liquid Drop Model[8].

Figures 1-14 illustrate the plot of the calculated double alpha decay half-life versus the decay energy for the isotopes with atomic numbers ranging from 60 to 93. Nuclei with deformed structures typically have shorter half lives indicating the decay more rapidly. It should be mentioned that the parent and daughter nuclei are viewed in this study as spheroid, and keeping the emitted fragment as spherical. From the table 1, and figures it is clearly seen that the deformation parameters alter the half life of an atomic nucleus by reducing the height and width of the potential barrier. Hence, comparing our half life values CYEM(WOD) and CYEM(WP&D), it is evident that when the deformation factors include, the half life values decreases.

Moreover, we have studied the double alpha decay and beryllium decay comparison of ^{229}Pu , ^{230}Am , ^{232}Cm , ^{234}Bk , ^{236}Cf , ^{239}Es , ^{241}Fm , ^{245}Md nuclei using our CYE Model.

Table-2 illustrates a comparative analysis of the half-lives associated with double alpha decay and beryllium emission from ^{229}Pu , ^{230}Am , ^{232}Cm , ^{234}Bk , ^{236}Cf , ^{239}Es , ^{241}Fm , ^{245}Md isotopes using our model and is also compared with other available models. The logarithmic half-life ($\text{Log } T_{1/2}(\text{s})$) computations for ^{229}Pu , ^{230}Am , ^{232}Cm , ^{234}Bk , ^{236}Cf , ^{239}Es , ^{241}Fm , ^{245}Md isotopes are presented by utilizing our model in the fifth column of table-2. Columns 6 and 7 of table-2 depict the half-life values corresponding to the MGLD Model[14] and Universal Decay Law(UDL)[14] respectively. As the mass defect of ^8Be is slightly larger than the double alpha particles, slightly increased Q-value and shorter half-life for 2α than ^8Be radioactivity. The half-lives for double alpha decay are less than that of ^8Be emission and as a result double alpha decay is more probable than ^8Be emission and it is clearly observed from table-2 and from fig.15. Besides, the possibility of ^8Be emission is less since it is a weakly bound nucleus.

4. CONCLUSION:

Double alpha decay (Parent Nucleus \longrightarrow Daughter Nucleus + 2α), is studied in this work using our Cubic plus Yukawa plus Exponential Model in the atomic number range from $Z=60$ to 93 and the results are compared with available theoretical model results, here deformation effects of parent and daughter nuclei are taken into account. Our results are in good agreement with other models. The deformation of a nucleus can significantly impact its half-life by altering the height and width of the potential barrier. From the table-1 and figures(1-14), it is seen that the half-life values are decreased because of considering deformation parameters. Thus, the nuclei with deformed structure have shorter half lives indicates decay more rapidly. Additionally, more dominant decay mode between double alpha decay and ^8Be decay is studied and the results are presented in table-2 and in fig.15, for ^{229}Pu , ^{230}Am , ^{232}Cm , ^{234}Bk , ^{236}Cf , ^{239}Es , ^{241}Fm , ^{245}Md isotopes using our CYE Model. As the mass defect of ^8Be is slightly larger than double alpha particles, the double alpha decay is more probable than ^8Be emission. The current findings provide valuable information and insights for refining theoretical models and guiding future experimental studies on double alpha decay.

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