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Ggβ*- Closure And Ggβ*- Interior In Grill Topological Spaces

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This research paper aims to introduce and investigate the characterizations of Ggβ*- closure and Ggβ*- interior by Ggβ*-closed set in Grill topological space. Some of the properties of them are obtained.

Keywords: Ggβ*-closed set, Ggβ*-open, Ggβ*- closure, Ggβ*- interior
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1. INTRODUCTION

The idea of grills on a topological space was first introduced by Choquet [4]. The concept of grills has shown to be a powerful supporting and useful tool. In [8], Hatir and Jafari have defined new classes of sets in a grill topological space and obtained a new decomposition of continuity in terms of grills. Ahmad Al-Omari and Takashi Noiri [1] introduced and investigated the notions of Gα-open sets, G-semi-open sets and Gβ-open sets in grill topological spaces. P. Gomathi Rajakumari and Jessie Theodore [7] has introduced Ggβ*- closed sets and studied its properties in Grill topological space. The aim of this Paper is to introduce and investigate the characterizations of Ggβ*- closure and Ggβ*- interior by Ggβ*-closed set in Grill topological space. Some of the properties of them are obtained.

2. PRELIMINARIES

Definition 2.1: [4] A non-null collection G of subsets of X is said to be a grill on X if

(i) $\phi \notin G$

(ii) $A \in G$ and $A \subseteq B$ implies that $B \in G$,

(iii) $A, B \subseteq X$ and $A \cup B \in G$ implies that $A \in G$ or $B \in G$.

Definition 2.2: [7] Let (X, τ) be a topological space and G be a grill on X. A mapping $\Phi_{G\beta}: P(X) \to P(X)$ is defined by $\Phi_{G\beta}(A) = \{x \in X : U \cap A \in G, \text{for all } U \in G\beta O(x)\}$ for each $A \in P(X)$. The mapping $\Phi_{G\beta}$ is called the Gβ-operator associated with grill G and the topology

τ and the family of semi-preopen sets $G\beta O(x) = \{U \in G\beta O(X) : x \in U\}$. Also the map $\Psi_{G\beta}: P(X) \to P(X)$ is defined by $\Psi_{G\beta}(A) = A \cup \Phi_{G\beta}(A)$ for all $A \in P(X)$. The map $\Psi_{G\beta}$ is called the Gβ-closure operator associated with grill G and the topology τ.

Definition 2.3: [7] Let (X,τ) be a topological space and G be a grill on X. Then a subset A of X is said to be a $Gg\beta^*$ closed if $\Phi_{G\beta}(A) \subseteq \text{int } U$ whenever $A \subseteq U$ and U is ω open in (X, τ). We denote the family of all $Gg\beta^*$ closed sets by $Gg\beta^*C(X, \tau)$.

Proposition 2.4: [7] Every G- semi closed set, Gα - closed set, G- closed set, β^*closed set, *g- closed set, g*- closed set, $g^{\#}s$- closed set, $g^{\#}$- closed set, Ggb closed set, Gg* closed set, b^*g closed set are $Gg\beta^*$ closed set but not conversely.

Theorem 2.5: [7] Assume that (X, τ, G) be a Grill topological space and A and B be any two $Gg\beta^*$closed sets. Then $A \cup B$ is $Gg\beta^*$closed.
Proposition 2.6: [7] For each x ∈ X, for each {x} is ω- closed or $\{x\}^c$ is $Gg\beta^*$-closed in X.

3. Ggβ*- OPEN SETS IN GRILL TOPOLOGICAL SPACES

Definition 3.1: Let (X, τ) be a topological space and G be a grill on X. Then a subset A of X is said to be a $Gg\beta^*$-open if A^c is $Gg\beta^*$-closed set in (X, τ, G). The family of all $Gg\beta^*$-open sets denoted by $Gg\beta^*O(X, \tau, G)$.

For example: Let X = {a, b, c}, τ = {ϕ, {a}, {a, c}, X} and G = {{a}, {a, b}, {a, c}, X}. Then $Gg\beta^*O(X, \tau, G) = \{\phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}, X\}$.

Theorem 3.2: In a Grill topological space, intersection of any two $Gg\beta^*$-open sets is $Gg\beta^*$-open.
Proof: Assume that A and B are $Gg\beta^*$-open sets. By theorem 2.5 and $(A \cap B)^c = A^c \cup B^c$, $(A \cap B)^c$ is $Gg\beta^*$-closed. Hence, $A \cap B$ is $Gg\beta^*$-open.

Remark 3.3: The union of two $Gg\beta^*$-open sets need not be $Gg\beta^*$-open.
For example: $X = \{a, b, c\}, \tau = \{\phi, \{c\}, X\}$ and $G = \{\{c\}, \{a, c\}, \{b, c\}, X\}$. Here the sets $A = \{a\}, B = \{b\}$ are $Gg\beta^*$-open sets but $A \cup B = \{a, b\}$ is not $Gg\beta^*$-open set.

Proposition 3.4: Every G- semi open set (respectively Gα - open set, G- open set, β^*- open set, Ggb-open and Gg*- open sets) is $Gg\beta^*$- open set but not conversely.

Proposition 3.5: Every $Gg\beta^*$-open sets is Ggsp-open (respectively, $G\hat{\eta}^*$- open) set.

Theorem 3.6: Let (X, τ, G) be a grill topological space and $A \subseteq X$. Then A is $Gg\beta^*$-open if and only if $F \subseteq \text{int}(\Phi_{G\beta}(A))$ whenever F is ω-closed and A is Gβ-open such that $F \subseteq A$.

Proof: Suppose that A is $Gg\beta^*$-open. Let F be ω-closed and A is Gβ-open such that $F \subseteq A$. Then $X - A \subseteq X - F$. Since A is Gβ-open, $\Phi_{G\beta}(X - A) \subseteq X - A$. Hence $\Phi_{G\beta}(X - A) \subseteq int\,(X - F)$.Therefore, $F \subseteq X - cl\left(\Phi_{G\beta}(X - A)\right) \subseteq X - cl\left(X - \Phi_{G\beta}(A)\right) = int\left(\Phi_{G\beta}(A)\right)$.

Conversely, suppose that $F \subseteq int(\Phi_{G\beta}(A))$ whenever F is ω-closed and A is Gβ-open such that $F \subseteq A$. Let U be ω-open set such that $X - A \subseteq U$. Then $X - U \subseteq A$ and $X - U \subseteq int(\Phi_{G\beta}(A))$. Also, $X - \Phi_{G\beta}(A) \subseteq X - cl(X - U) = int\,(U)$. Therefore, $\Phi_{G\beta}(X - A) \subseteq int(U)$. Hence, A is $Gg\beta^*$-open.

Corollary 3.7: Let (X, τ, G) be a grill topological space and $A \subseteq X$. Then A is $Gg\beta^*$-open if and only if $F \subseteq int(\Phi_{G\beta}(A))$ whenever F is closed and A is $G\beta$-open such that $F \subseteq A$.
Proof: Every closed set is ω-closed. Then the proof is similar.

Theorem 3.8: Let (X, τ, G) be a grill topological space and $A \subseteq X$. Then A is $Gg\beta^*$-open if and only if $U = X$ whenever U is ω-open such that $\Phi_{G\beta}(A) \cup A^c \subseteq U$ and (X, τ) be a T_ω-space.
Proof: Let A be a $Gg\beta^*$-open and U be ω-open such that $\Phi_{G\beta}(A) \cup A^c \subseteq U$. This implies that $U^c \subseteq \left(\Phi_{G\beta}(A) \cup A^c\right)^c = \left(\Phi_{G\beta}(A)\right)^c \cap (A^c)^c = \Phi_{G\beta}(A^c) - A^c$. Since A^c is $Gg\beta^*$-closed and U^c is ω-closed and by theorem 2.3.57, $U^c = \phi$ implies that $U = X$.

Conversely, suppose that F is ω-closed and $F \subseteq A$. Then, $\Phi_{G\beta}(A) \cup A^c \subseteq \Phi_{G\beta}(A) \cup F^c$. Since $\Phi_{G\beta}(A)$ and F^c are ω-open, then $\Phi_{G\beta}(A) \cup F^c$ is ω-open. Hence by hypothesis $\Phi_{G\beta}(A) \cup F^c = X$. This implies $F \subseteq \Phi_{G\beta}(A) \subseteq int(\Phi_{G\beta}(A))$, since every ω-closed set is closed in a T_ω- space. Thus, A is $Gg\beta^*$-open**.**

Theorem 3.9: Assume that (X, τ, G) be a Grill topological space and A be a $Gg\beta^*$-open in X. If B is a subset of X such that $\Phi_{G\beta}(A) \subseteq B \subseteq A$, then B is $Gg\beta^*$-open**.**
Proof: Let A be a $Gg\beta^*$-open and U be ω-open set such that $B^c \subseteq U$. Since $\Phi_{G\beta}(A) \subseteq B \subseteq A$, then $A^c \subseteq B^c \subseteq \left(\Phi_{G\beta}(A)\right)^c$. Also $A^c \subseteq U$ and A^c is $Gg\beta^*$-closed. Therefore, by theorem 2.3.62, B^c is $Gg\beta^*$-closed implies that B is $Gg\beta^*$-open.

Proposition 3.10: Assume that (X, τ, G) be a Grill topological space and $A \subseteq X$. If A is $Gg\beta^*$-open, then $\Phi_{G\beta}(A) - A$ is $Gg\beta^*$-closed but not conversely.
Proof: Suppose that A is $Gg\beta^*$-open. Let U be ω-open set such that $\Phi_{G\beta}(A) - A \subseteq U$. Then, $\Phi_{G\beta}\left(\Phi_{G\beta}(A) - A\right) \subseteq \Phi_{G\beta}(A) - A \subseteq U = int(U)$. Hence, $\Phi_{G\beta}(A) - A$ is $Gg\beta^*$-closed.
The converse is not true by the following example.

Example 3.11: Let $X = \{a, b, c, d\}, \tau = \{\phi, \{a\}, X\}$ and $G = \{\{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, X\}$. For the set $A = \{c, d\}$, $\Phi_{G\beta}(A) - A = \phi$ which is $Gg\beta^*$-closed but $\{c, d\}$ is not $Gg\beta^*$-open.

4. $Gg\beta^*$- CLOSURE AND $Gg\beta^*$- INTERIOR

Definition 4.1: Let $A \subset X$. The $Gg\beta^*$-closure of A is defined as the intersection of all $Gg\beta^*$-closed sets containing A and denoted by $Gg\beta^*cl(A)$. That is $Gg\beta^*cl(A) = \cap \{V: A \subseteq V, V \in Gg\beta^*C(X,\tau)\}$ where $Gg\beta^*C(X,\tau)$ is the family of all $Gg\beta^*$- closed sets in X.

For example: Let $X = \{a,b,c\}; \tau = \{\phi,\{a\},\{a,b\},X\}$ and $G = \{\{a\},\{a,b\},\{a,c\},X\}$. Then $Gg\beta^*C(X,\tau) = \{\phi,\{b\},\{c\},\{a,c\},\{b,c\},X\}$.

A	{a}	{b}	{c}	{a, b}	{a, c}	{b, c}	X	ϕ
cl(A)	X	{b, c}	{c}	X	X	{b, c}	X	ϕ
$Gg\beta^*cl(A$	{a, c}	{b}	{c}	X	{a, c}	{b, c}	X	ϕ

Table 4.1: $Gg\beta^*$- Closure of subsets of X

Lemma 4.2: For any $A \subseteq X$, $A \subseteq Gg\beta^*cl(A) \subseteq cl(A)$.
Proof: Since every closed set is $Gg\beta^*$-closed, therefore $Gg\beta^*cl(A) \subseteq cl(A)$.

Lemma 4.3: If $A \subset B$, then $Gg\beta^*cl(A) \subseteq Gg\beta^*cl(B)$.
Proof: Follows from the Definition

Remark 4.4: The $Gg\beta^*$-closure of a set need not be $Gg\beta^*$-closed set by the following example.

Example 4.5: Let $X = \{a,b,c,d\}$; $\tau = \{\phi,\{b,c\},X\}$ and $G = \{\{a,c\}, \{b,c\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, X\}$. For the set $A = \{b,c\}, Gg\beta^*cl(A) = \{b,c\}$ but $\{b,c\}$ is not $Gg\beta^*$-closed.

Proposition 4.6: Let (X, τ, G) be a grill topological space. If A is $Gg\beta^*$-closed set, then $Gg\beta^*cl(A) = A$ but not conversely.
Proof: The proof follows from the definition.
The converse of this proposition is not true by the following example.

Example 4.7: Let $X = \{a,b,c,d\}; \tau = \{\phi,\{b,c\},X\}$ and $G = \big\{\{a,c\}, \{b,c\},\{a,b,c\}, \{a,c,d\}, \{b,c,d\}, X\big\}$. Here $Gg\beta^*cl(\{b,c\}) = \{b,c\}$ but $\{b,c\}$ is not $Gg\beta^*$-closed.

Theorem 4.8: Let (X, τ, G) be a grill topological space. Then $Gg\beta^*$-closure is a Kuratowski closure operator on X.

Proof:

(i) Since ϕ and X are $Gg\beta^*$-closed, then $Gg\beta^*cl(\phi) = \phi$ and $Gg\beta^*cl(X) = X$.
(ii) By lemma 4.3, $Gg\beta^*cl(A) \subseteq Gg\beta^*cl(B)$.

(iii) Assume that $M, N \subseteq X$. By lemma 4.3, we get $Gg\beta^*cl(A) \subseteq Gg\beta^*cl(A \cup B)$ and $Gg\beta^*cl(B) \subseteq Gg\beta^*cl(A \cup B)$. Therefore, $Gg\beta^*cl(A) \cup Gg\beta^*cl(B) \subseteq Gg\beta^*cl(A \cup B)$. If $x \notin Gg\beta^*cl(A) \cup Gg\beta^*cl(B)$ then there exit $U, V \in Gg\beta^*C(X, \tau)$ such that $A \subset U, x \notin U, B \subset V$ and $x \notin V$. Hence, $A \cup B \subset U \cup V$. By theorem 2.5, $U \cup V$ is $Gg\beta^*$-closed. Thus $x \notin Gg\beta^*cl(A \cup B)$. Hence, $Gg\beta^*cl(A \cup B) \subseteq Gg\beta^*cl(A) \cup Gg\beta^*cl(B)$. Thus, $Gg\beta^*cl(A) \cup Gg\beta^*cl(B) = Gg\beta^*cl(A \cup B)$.

(iv) Let $U \subseteq X$ and A be a $Gg\beta^*$-closed set containing U. By Definition 4.3, $Gg\beta^*cl(U) \subset A$ and $Gg\beta^*cl(Gg\beta^*cl(U)) \subset Gg\beta^*cl(A) = A$. This implies $Gg\beta^*cl(Gg\beta^*cl(U)) \subset \cap \{A: U \subset A,\ A \in Gg\beta^*C(X, \tau)\} = Gg\beta^*cl(U)$. Also $Gg\beta^*cl(U) \subset Gg\beta^*cl(Gg\beta^*cl(U))$. Hence, $Gg\beta^*cl(U) = Gg\beta^*cl(Gg\beta^*cl(U))$.

Thus, $Gg\beta^*$-closure is a Kuratowski closure operator on X.

Definition 4.9: The topology on X generated by $Gg\beta^*$-closure is defined as

$$\tau_{Gg\beta^*} = \{A: Gg\beta^*cl(A^c) = A^c\}.$$

Proposition 4.10: In a grill topological space (X, τ, G), $\tau_{Gg\beta^*}$ is a topology for X.
Proof: The proof is follows from the theorem 4.8.

Proposition 4.11: Let A be a subset of a grill topological space (X, τ, G). For any $x \in X, x \in Gg\beta^*cl(A)$ if and only if $U \cap A \neq \phi$ for every $Gg\beta^*$-open set U containing x.
Proof: Suppose that $x \in Gg\beta^*cl(A)$. Let U be a $Gg\beta^*$-open set containing x such that $U \cap A = \phi$. This implies that $A \subset U^c$. But U^c is $Gg\beta^*$-closed and hence $Gg\beta^*cl(A) \subset U^c$. Since $x \notin U^c$, then $x \notin Gg\beta^*cl(A)$ which is contradiction to the hypothesis.

Conversely, assume that every $Gg\beta^*$-open set containing x such that $U \cap A = \phi$. If $x \notin Gg\beta^*cl(A)$, then there exist a $Gg\beta^*$-closed set F such that $A \subset F$ and $x \notin F$. This implies that $F^c \cap A = \phi$. Therefore, $x \in F^c$ and F^c is an $Gg\beta^*$-open set containing x such that $F^c \cap A = \phi$ which is contradiction to hypothesis. Hence, $x \in Gg\beta^*cl(A)$.

Definition 4.12: For any $A \subset X$, $\boldsymbol{Gg\beta^*}$**- interior** of A is defined as the union of all $Gg\beta^*$-open sets contained in A and denoted by $Gg\beta^*$int(A). That is $Gg\beta^*int(A) = \cup \{U: U \subseteq A, U \in Gg\beta^*O(X, \tau)\}$.

For example: Let $X = \{a, b, c\}$; $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $G = \{\{a\}, \{a, b\}, \{a, c\}, X\}$. Then $Gg\beta^*O(X, \tau) = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$.

A	{a}	{b}	{c}	{a, b}	{a, c}	{b, c}	X	ϕ
int (A)	{a}	ϕ	ϕ	{a, b}	{a}	ϕ	X	ϕ

$Gg\beta^* int(A)$	{a}	{b}	ϕ	{a, b}	{a, c}	{b}	X	ϕ

Table 4.2: $Gg\beta^*$- interior of subsets of X

Proposition 4.13: For any $A, B \subseteq X$,

(i) $int(A) \subseteq Gg\beta^* int(A)$.

(ii) If $A \subset B$, then $Gg\beta^* int(A) \subseteq Gg\beta^* int(B)$.

(iii) $Gg\beta^* int(A) \cup Gg\beta^* int(B) \subseteq Gg\beta^* int(A \cup B)$.

Proof: Follows from definition.

Proposition 4.14: If A is $Gg\beta^*$-open, then $A = Gg\beta^* int(A)$.
Proof: Obvious.
Converse of this Proposition is not true. It is seen by the following example.
Example 4.15: Let $X = \{a,b,c,d\}; \tau = \{\phi, \{b,c\}, X\}$ and $G = \{\{a,c\}, \{b,c\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, X\}$. Here $Gg\beta^* int(\{a\}) = \{a\}$ but $\{a\}$ is not $Gg\beta^*$-open.

Proposition 4.16: Let A be a subset of a space X. Then the following are true

(i) $(Gg\beta^* int(A))^c = Gg\beta^* cl(A^C)$

(ii) $Gg\beta^* int(A) = (Gg\beta^* cl(A^C))^c$

(iii) $Gg\beta^* cl(A) = (Gg\beta^* int(A^C))^C$

Proof:

(i) Let $x \in (Gg\beta^* int(A))^c$. Then $x \notin Gg\beta^* int(A)$. That is every Ggβ*-open set U containing x is such that $U \not\subset A$. Thus every $Gg\beta^*$-open set U containing x is such that $U \cap A^C \neq \phi$. By Proposition 4.11, $x \in Gg\beta^* cl(A^C)$ and therefore $(Gg\beta^* int(A))^c \subset Gg\beta^* cl(A^C)$. Conversely, let $x \in Gg\beta^* cl(A^C)$. Then by Proposition 4.11, every $Gg\beta^*$-open set U containing x is such that $U \cap A^C \neq \phi$. By definition 4.14, $x \notin Gg\beta^* int(A)$. Hence $x \in (Gg\beta^* int(A))^c$ and so $Gg\beta^* cl(A^C) \subset (Gg\beta^* int(A))^c$. Hence $(Gg\beta^* int(A))^C = Gg\beta^* cl(A^C)$.

(ii) Follows by taking complements in (i).

(iii) Follows by replacing A by A^C in (i).

Proposition 4.17: For a subset A of a grill topological space (X,τ, G), then the following conditions are equivalent.

(i) $Gg\beta^* O(\tau)$ is closed under any union.

(ii) A is $Gg\beta^*$-closed if and only if $Gg\beta^* cl(A) = A$.

(iii) A is $Gg\beta^*$-open if and only if $Gg\beta^* int(A) = A$.

Proof:

(i) ⇒(ii): Let A be a $Gg\beta^*$-closed set. Then by the definition of $Gg\beta^*$-closure we get $Gg\beta^* cl(A) = A$. Conversely, assume $Gg\beta^* cl(A) = A$. For each $x \in A^c$ and $x \notin Gg\beta^* cl(A)$, by Proposition 4.11, there exists a $Gg\beta^*$-open set F_x containing x such

that $F_x \cap A = \phi$ and hence $x \in F_x \subset A^c$. Therefore we obtain $A^c = \cup_{x \in A^c} F_x$.
By (i) A^c is $Gg\beta^*$-open and hence A is $Gg\beta^*$-closed.
(ii) ⇒ (iii): Follows by (ii) and Proposition 4.16.
(iii) ⇒ (i): Let $\{U_\alpha / \alpha \in \Lambda\}$ be a family of $Gg\beta^*$-open sets of X. Put $U = \cup_\alpha U_\alpha$. For each $x \in U$, there exists $\alpha(x) \in \Lambda$ such that $x \in U_{\alpha(x)} \subset U$. Since $U_{\alpha(x)}$ is $Gg\beta^*$-open, $x \in Gg\beta^* int(U)$ and so $U = Gg\beta^* int(U)$. By (iii), U is $Gg\beta^*$-open. Thus $Gg\beta^* O(\tau)$ is closed under any union.

Proposition 4.18: In a grill topological space (X, τ, G), assume that $Gg\beta^* O(\tau)$ is closed under union. Then $Gg\beta^* cl(A)$ is a $Gg\beta^*$-closed set for every subset A of X.
Proof: Since $Gg\beta^* cl(Gg\beta^* cl(A)) = Gg\beta^* cl(A)$ and by Proposition 4.17, $Gg\beta^* cl(A)$ is a $Gg\beta^*$-closed set.

5. VARIOUS SPACES BY Ggβ*-CLOSED SETS

Definition 5.1: A space X is called a $T_{Gg\beta^*}$ space if every $Gg\beta^*$-closed set is closed.

Example 5.2: Let $X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{a, \{b, c\}, X\}$ and $G = \{\{b\}, \{a, b\}, \{b, c\}, X\}$. Then X is a $T_{Gg\beta^*}$-space.

Definition 5.3: A space X is called a ${}_{sp}T_{Gg\beta^*}$ space (respectively ${}_{p}T_{Gg\beta^*}$, ${}_{s}T_{Gg\beta^*}$, ${}_{\alpha}T_{Gg\beta^*}$) if every $Gg\beta^*$-closed set is semi- pre closed (respectively pre closed, semi closed, α - closed).

Proposition 5.4: If X is a $T_{Gg\beta^*}$-space, then every singleton of X is either Gω- closed or open.
Proof: By the hypothesis and by Proposition 2.6, the proof follows. The converse of Proposition 5.4 is not true.

Example 5.5: Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $G = \{\{b\}, \{a, b\}, \{b, c\}, X\}$. Then the space X satisfies the conclusion of Proposition 5.4, but X is not a $T_{Gg\beta^*}$ space since {b} is β*-closed but not closed.

Definition 5.6: A space X is called a $T_{G\beta}$-space if every Gβ-closed set is closed.

Proposition 5.7: Every $T_{G\beta}$ space is a $T_{Gg\beta^*}$ space but not conversely.
Proof: Let X be a $T_{G\beta}$- space. Let A be any Ggβ*-closed set in X. Since every Ggβ*-closed set is a Gβ - closed set and X is a $T_{G\beta}$ - space, A is closed.
Example 5.8: Let $X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $G = \{\{b\}, \{a, b\}, \{b, c\}, X\}$. Then X is a $T_{Gg\beta^*}$ space but not a $T_{G\beta}$-space, since the set $B = \{b\}$ is Gβ - closed but not closed.

Definition 5.9: A space X is called a ${}_{G\alpha}T_{Gg\beta^*}$ (resp. ${}_{Gp}T_{Gg\beta^*}$, ${}_{G\beta}T_{Gg\beta^*}$, ${}_{Gs}T_{Gg\beta^*}$) if every Ggβ*- closed set is Gα-closed (resp. G-preclosed, G-semi-preclosed and G-semi-closed).

Proposition 5.10: Every $T_{Gg\beta^*}$ space is ${}_{G\alpha}T_{Gg\beta^*}$, ${}_{Gp}T_{Gg\beta^*}$, ${}_{G\beta}T_{Gg\beta^*}$ and ${}_{Gs}T_{Gg\beta^*}$ space.
Proof: Since every $Gg\beta^*$-closed set is closed in a $T_{Gg\beta^*}$- space and every closed set is Gα-closed, G-pre-closed, Gβ-closed, G-semi closed, the proof follows.

Theorem 5.11: In a grill topological space X, the following conditions are equivalent
(i) X is a ${}_{G\beta}T_{Gg\beta^*}$ space.
(ii) Every singleton of X is either Gω-closed or Gβ-open.

Proof: Let $x \in X$ and suppose that $\{x\}$ is not Gω-closed in X. Then $\{x\}^C$ is not Gω-open in X. Since X is the only Gω-open set containing $\{x\}^C$, and $G\beta(\{x\}^C) \subset X = \text{int } X, \{x\}^C$ is $Gg\beta^*$-closed. By (i) $\{x\}^C$ is G-semi pre closed and hence $\{x\}$ is Gβ-open. Conversely, let A be a $Gg\beta^*$-closed set in X. Clearly $A \subset G\beta cl(A)$. Let $x \in Gspcl(A)$, by (ii), $\{x\}$ is either Gω-closed or Gβ-open. Then there are two cases.
Case (i): Suppose that $\{x\}$ is Gω-closed and if $x \notin A$, then $G\beta cl(A) - A$ contains the Gω-closed set $\{x\}$. But A is $Gg\beta^*$-closed. This is a contradiction. Thus $x \in A$.
Case (ii): Suppose that $\{x\}$ is G semi-preopen, since $x \in G\beta cl(A)$; $\{x\} \cap A \neq \phi$. So $x \in A$. Hence in both the cases, $x \in G\beta cl(A)$ implies $x \in A$. Hence $G\beta cl(A) \subset A$ which implies A is Gβ-closed.

6. CONCLUSION

In this article we have introduced and investigated the characterizations of $Gg\beta^*$-closure and $Gg\beta^*$- interior by $Gg\beta^*$-closed set in Grill topological space and the operator $Gg\beta^* cl(\tau)$ is a Gβ-Kuratowski closure operator. Also, we have defined and investigated the various spaces by $Gg\beta^*$closed sets and some of the properties of them are obtained.

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