



Convolution Maximization Sequence in $L_{1+\epsilon}$ (\mathbb{R}^n) for Kernels of Lorentz Space

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Convolution maximization sequences in the (\mathbb{R}^n) kernel of Lorentz space have garnered considerable attention because of their theoretical importance and practical implications in various fields such as signal processing machine learning and mathematical analysis. We study the properties and applications of convolution maximization sequences In Lorentz spaces exploring their convergence behavior stability and computational efficiency. We study the interplay between kernels of the Lorentz space and convolution optimization providing insights into the geometric and functional aspects of function space geometry. Through theoretical analysis numerical simulations and experimental validations we demonstrate the effectiveness and applicability of convolution maximization sequences in diverse domains from image processing to differential equations. Our study contributes to the development of the understanding of convolution optimization and function space geometry and offers new perspectives on optimization theory and mathematical analysis. Moreover we discuss possible future research directions and highlight the broader significance of the sequences of a convolution maximization in contemporary mathematics and computational science.

Keywords: Convolution Maximization Sequences, Lorentz Spaces, Optimization Theory, Signal Processing.

1. Introduction

Convolution maximization sequences attention-based algorithms prove extremely useful in signal processing or machine learning, and harmonic analysis. It should be noted that particularly in the case of function spaces under Lorentz norms that such sequences become even more important because of their similar applications in the investigation of the behavior of the functions themselves as well as operations being conducted upon them. The real number space denoted by (\mathbb{R}^n) is a Banach space possessing Lorentz sill which is greater than or equal to those familiar norms. Indeed this space would already belong to the settings of partial differential equations and exterior calculus, and optimal transport.

The question about mean values of convolution integrals on $L_{1+\epsilon}$ (\mathbb{R}^n) for kernels space–Lorentz continues to be hot recently. Here we discuss the teachers for both their theoretical

and practical benefit as well for teacher who may study their character and behavior as the approximation theory, data analysis and function space geometry.

In the current paper we considerably reasons out the convolution growth sequence for (Rn) series of functions from the Lorentzk spaces particularly. Obtaining convolution maximization sequences remains a problem that has not yet been solved. While significant progress has been attained in frequency domain analysis, optimization, and spectral estimation, an organized review of the sequence or convergence properties as well as applications of convolution maximization sequences is the main focus of this section. You are in a good position to achieve this target after evoking a blend of techniques from the research materials which are available. For example, metahorical methods are used from calculation optimization (Genevay et al., 2016), dynamical spectrum analysis (Braverman et al., 2021) and wave analysis (Taheri, 2015). Moreover, they start their examination using the recent results obtained with the optimal transport (Feydy et al., 2019; Gialani, 2021) and the machine learning (Han et al., 2020; Han et al., 2021) that make to make their study comprehensive.

The remainder of this paper is organized as follows: in Sect. Two of this topic we investigate the QFT of the convolution cyclic sequences and Lorentz space kernels, respectively. In this subsection (part 3), we lay down the mathematical preliminaries, that are needed to perform our analysis. Part number 4 is devoted to the reproduction /maximization patterns and the formula for convolution maximization sequences. The final part of the paper introduces the usage of kernels in the Lorenz space. On the Section 6 we show via performance results and numerical simulations the argument we stand for. The final segment of my research paper is all about the outcomes. It includes a detailed analysis of the findings and the ways we could take it to the next level. Thus, in Section 8, we review the paper briefly with the conclusion knot for the main findings and focused points covered in the different parts of the paper.

The paper is working on a description of interplay between accumulating sequences for convolution and Lorentz spacetime kernels which is likely to contributory to function space geometry knowledge and to give theoretical hints useful in various applications.

2. BACKGROUND AND RELATED WORK

Here we present a detailed account of the background materials that focus on the convolutional maximization sequences and the Lorentz kernels in the space. On as well we apply to the existing relevant literature which has made added knowledge to these subjects.

Convolution Maximization Sequences

Convolution maximization sequences are sequences of functions that maximize these convolution integrals. These are over a given domain. These formulae have been thoroughly investigated in the majority of function spaces, including Lebesgue spaces, Sobolev spaces, as well as the most recent Lorentz spaces. The convolution maximization sequences belong to the convolution properties similar to convolution operator behavior and its associated kernels. Previous studies about the convergence process of maximum sequence convolution structures studied their properties of convergence and their existence and

uniqueness. Altschuler et al. (2019) introduced the massively scalable Sinkhorn distance via Nyström method, namely, fast algorithms for optimal transport plans and related quantities. Genevay et al. (2016) made an attempt to find the answers to the computational aspects of such large-scale optimal transport problems using stochastic optimization techniques.

Also, Sadov (2022) studied the existence of convolution maximizers for kernels from Lorentz spaces in $Lp(n)$. She showed the implication of the convolution optimization process with respect to function space structure. These books together let us comprehend the convolution maximization sequences and when their applications fit.

Lorentz Space Kernels

Lorentz spaces, labeled $L_{1+\epsilon}(\mathbb{R}^n)$, are function spaces endowed with a norm defined by a weight function that is positive and not equal to zero. These spaces are the generalizations of the familiar Lebesgue spaces, and have therefore been applied to operations in harmonic analysis, PDEs and mathematical physics. The core structures in Lorentz spaces have a significant implication in the definition of convolution operators and the analysis of integral transforms.

In this regard, one of the most recent breakthroughs in harmonic analysis is related to the study of Lorentz spaces kernels and the features and conduct of the same. Bennett and Bez (2022) dealt with higher-order transversality in the harmonic analysis. They have discovered important kernels as well as their link with the geometric structure of the function spaces. Moreover, in (Rielly et al. 2023) the researchers looked into learning high-dimensional nonparametric differential equations via occupation kernel functions, indicating the importance of kernel-based methods in the data-driven modeling and analysis. The underlying kernels of Lorentz spaces are key in the assessment of the performance of convolution maximization sequences and their potential use. Through utilizing approaches of harmonic analysis, optimization, and machine learning, researchers have been able to draw fundamental properties and possible use cases for Lorentz spreading kernels.

In the sections that follow, we use this background to formalize and investigate convolution maximizing sequences inscribing them in Lorentz spaces which hold the point-wise multipliers. We consider the interrelation between the convolution optimization and the functional space geometry through theoretical analysis, numerical simulations, and experimental validations, which aimed to deepen our understanding of this interplay.

Table 1 hereunder is a resume of current related research.

Table1: Comprehensive Table of Related Works

Authors	Year	Works and Results
Altschuler et al	2019	Authors develop a novel approach to compute Sinkhorn distances at the noscale using the Nyström method. The practice shows the ability of answer this task through solving complex transport tasks at the optimal level; Moreover, the obtained results can be applicable in machine learning and data analysis.

Bennett et al	2022	This preprint extends higher order transversality notions from harmonic analysis imparting a new flavor to the subject thus giving a deeper insight into the structure of function spaces and their role in mathematical analysis as well as PDEs.
Braverman et al	2021	Authors describe fast algorithms for determining the power spectrum of data sets and time series simulations in the time of order or suborder in practical aspect of the analysis of large data sets.
Breustedt et al	2023	This paper addresses the subset selection impact on 'informed' graph neural networks, which is the designing and optimising neural networks for graph data.
Chen et al	2020	The authors possess Stein–Weiss-type inequalities, the bigger order of which is created by a fractional Poisson kernel and thereby expend the classical counterparts along the same path.
Chen et al	2019	This paper deals with the Hardy—Littlewood —Sobolev inequalities with the fractional Poisson kernel constituting more evidence regarding the influence of the harmonic functions and their efficiency in the partial differential expression.
de Laat	2016	This doctoral dissertation aims at describing and searching moment inequalities in extremal geometry, delivering wide-range analysis to the optimization problems in engineering and mathematics.
Fang et al	2019	The authors proposed a method in which the quality features are learn deeply though diagnosis of breast cancer using images, and the result showed that the accuracy could be enhanced in a diagnostic way.
Feydy et al	2019	This report illustrates a method for smoothing between optimal transport and maximum mean discrepancy using Sinkhorn divergences, this implies an extension that provides a flexible apparatus for comparison and training of probability distributions.
Genevay et al	2016	Authors show stochastic optimization methods to tackle big-sized optimal transport problems, their approach is capable to handle such large-scale data, as indicated by efficiency and scalability.
Gilani	2021	This dissertation looks at the uses of diffusion maps with regards to time series prediction, filtering, and elliptic partial differential equations solving on the second order. These stimulate novel views on data modeling and analysis.
Grattarola et al	2019	The two authors propose a method for change detection through the learning of graph embeddings based on the constant-curvature manifold, and they show graph as a convenient tool to catch the anomalies and structural changes of dynamic graphs.
Han et al	2020	This work proposes a comprehensive learning system that aims at fetching the maximum possible information from the chaotic time series in large-scale. Accurate prediction of dynamical systems is improved with the methodology.
Han et al	2021	The authors first present GLocal-K, an approach that uses global and local kernels to widen the range of items recommending and optimize performance of the models under various contexts.

Kunzinger et al	2022	We elaborated the comparison of synthetic and distributional lower Ricci curvature bounds in this preprint. The discussed geometric properties of the metric measure spaces are crucial to explain their basic possibilities for differential geometry and analysis.
Lauretta et al	2017	This article elaborates on OSIRIS-REx, a spacecraft mission to acquire samples from Bennu asteroid and deliver them to Earth for laboratory analysis which would be instrumental in learning more about early solar system.
Ong et al	2019	The authors suggest providing or approximating the local operators that may function as a Bayesian regularization as well as proximal operators, that enable to solve image denoising, deblurring and inverse problems in imaging efficiently.
Ramis	2023	The main aim of my bachelor thesis is the exploration of the modern harmonic analysis by considering the singular integrals, maximal functions, and the Littlewood-Paley theory. The last part of the thesis will be the review of the harmonic analysis tools..
Rielly et al	2023	The present work proposes multivariable occupation kernel functions which is used for parameterizing high dimensional nonparametric differential equation for computationally efficient implementation. The developed framework permits modeling of complex dynamical systems and data-driven discovery.
Sadov	2022	The given preprint carries out the discussion about the existence of convolution maximizers in spaces from Lorentz spaces to a kernels from Lurenz spaces with subsequent insights into the optimization properties with intensity of convolution operators applied to functions.
Saini	2023	This paper presents a method of solar aurora forecast from imbalanced multivariate time series data using random convolutional transform kernel of minor type, which is the alternative for astronauts and space weather modellers.
Scholkopf et al	2018	In this book kernel methods are given a systematic introduction that covers array of machine learning tools including support vector machines, regularization, optimization, and application of them in various fields.
Sobczyk et al	2022	The authors propound a technique of spectral density reconstruct using the Chebyshev polynomials which has better robustness and computational efficiency for deducing the spectral features from empirical data.
Taheri	2015	Whether studying function spaces, synthetic geometry, or mathematics in general, this book will offer students an advanced perspective on the mathematical analysis of differential equations and many other relevant topics.

Tolba et al	2022	This article harnesses a BLACK-BOX neural network attack to evaluate and assess multimedia P-BOX security. This new and modern approach is the step-stone for analyzing the resistance of multimedia encryption algorithms.
Yuan et al	2019	Authors suggest the use of a space-variant fast error-bounded bilateral filtering method to speed up service which can be applied conveniently in computer vision and image processing where enhancement is attained and noise reduction.
Zayed	2015	In this paper the energy concentration phenomena are addressed in variable reproducing-kernel Hilbert spaces (RKHS) which do not only give a perspective on the structure of kernel-based signal description spaces but also their application in signal processing and machine learning.

3. MATHEMATICAL PRELIMINARIES

Within this area, we give the needed mathematical history and also overviews for our evaluation of merging optimization series in $L^{1+\epsilon}(\mathbb{R}^n)$ for bits of Lorentz area.

$L^{1+\epsilon}(\mathbb{R}^n)$ Space

Lorentz areas, indicated as $L^{1+\epsilon}(\mathbb{R}^n)$, are feature areas furnished with a standard specified by a non-negative weight feature. Formally for a quantifiable feature $f: \mathbb{R}^n \rightarrow \mathbb{C}$, the Lorentz standard is provided.

$$\|f\|_{+} = \left(\int_{\mathbb{R}^n} |f(x)| w(x) dx \right)^{1+\epsilon}, \dots\dots\dots (1)$$

where $w(x)$ is a non-negative weight function, and (ϵ) exceeds 0 revealing the work ability of the norm. Lorentz spaces are a flexible generalization of both Levenberg-Marquardt spaces and spaces of rearrangement-invariant functions, which can be used to study a variety of properties of function spaces.

Convolution in $L^{1+\epsilon}(\mathbb{R}^n)$

Convolution is a fundamental operation in functional analysis, particularly in the context of integral transforms and linear operators. Given two functions $f, g: \mathbb{R}^n \rightarrow \mathbb{C}$, their convolution $f * g$ is defined as:

$$(f * g)(x) = \int_{\mathbb{R}^n} f(y) \cdot g(x - y) dy \dots\dots\dots (2)$$

In \mathbb{R}^{1+n} the convolution operator is modified so it keeps the specific norm structure of the space. This results in the fascinating characteristics and manifestations which are most pronounced while using convolution maximization sequences.

Properties and Characteristics of Lorentz Space

Lorentz spaces have various crucial features and properties that differentiate them from the

other functional spaces. These include:

Rearrangement-invariance

Lorentz spaces are necessarily rearrangement invariant with the norm being invariant under some functional rearrangements.

Interpolation properties

Lorentz spaces provide a means of interpolation between Lebesgue spaces, Sobolev spaces, and other function spaces, thus allowing us to study diverse phenomenon precisely.

Duality

The Lorentz spaces deal with the concept of duality in a natural manner, so functionals and dual spaces can be studied.

The properties of these sequences play a significant role in investigating and inspecting convolution maximization sequences for the sake of their behavior in Lorentz spaces. It is these mathematical preconditions that we will use in the next sections to establish and analyze filter maximization sequences in $L^{1+\epsilon}(\mathbb{R}^n)$ with kernels from Lorentz spaces in particular. With theoretical analysis, numerical simulations and experimental verifications we try to study the interrelations between the convolution optimization and function space geometry by deepening the understanding.

4. CONVOLUTION MAXIMIZATION SEQUENCE

This part is devoted to the design and properties of colligation maximization sequences in $Q_n + V \subseteq$ kernels of Lorentz space.

Definition and Formulation

In our goal concerning the maximizations of convolution integrals in the periodic space $A(\mathbb{R}^n)$, we intend to discover a sequence with elements that contribute their full potential within this domain. Firstly, we denote the functions sequence $f_n(x)$ in \mathbb{R}^d by $f_n(x)$ that is defined on $L^{1+\epsilon}$. And then a kernel function $k(x, y)$ is of our interest. The convolution maximization problem can be formulated as: The convolution maximization problem can be formulated as:

$$f_n^* = \arg \max \int_{\mathbb{R}^n} f_n(x) * f_n(x) dx \quad \dots\dots\dots (3)$$

The success of this Fourier transform technique crucially lies in the fact that it is difficult to find the f_n^* which gives the maximum value of integration. This is achieved only by using highly sophisticated optimization techniques and functional analysis tools.

Properties and Characteristics

Convolution maximization sequences exhibit several interesting properties and characteristics in $L^{1+\epsilon}(\mathbb{R}^n)$:

Convergence behavior

It is critical to know the convergence features of sequences used by convolution maximization for analyzing results for practical applications. In addition to this using the convergence rates, stability and convergence in distributions should also be considered.

Existence and uniqueness

The question of whether the existence and singularity of convolution maximizers for operators in \mathbb{R}^n -ONT are equivalent is a fundamental one for the development of both optimization theory and functional analysis.

Computational efficiency

The design of a wise algorithm for calculating convolution aggregation sequences is one of the main tasks for these classes of real-life problems involving signal processing, image analysis, and machine learning.

Applications

The convolutional maximizing solutions are applicable widely in several sectors, for example, for image denoising, signal enhancement, pattern recognition, among others. Acknowledging the theoretical concepts as well as the practical aspects of these kinds of sequences is a fundamental act to use them advantageously in real-world activities.

Theoretical Analysis

The convergence properties and stability, as well as the computational complexity of the maximization are examined in the theoretical analysis and a practical way of solving them mathematically is also given. This utilizes the equipment of functional analysis, optimization theory, and numerical analysis to obtain deep and exact discovery about the functioning of such sequences.

Theorems of convergency and algorithmical complexity have been discovered and proven in recent studies on maximization of the convolution sequence that are presently available. By bounded the theoretical base, we are going to contribute to progress in convolution maximization sequences in $\mathbb{L}^1 + \mathbb{R}e$ for kernels of c -dimensional space and this goes to be valuable aids for researchers and its consumer that seeks to exploit the potential opportunities in such realm.

In the latter sections one will surely find a discussion on the convergence properties of the MCS, numerical algorithms related to them and applications like sensor signals processing and machine learning. Our primary task is a theoretical inspection that will involve formulation of numerical simulations and real-world validations that will show the essence and potential use of convolution maximization sequences in the case of Lorentz space kernels.

5. KERNELS OF LORENTZ SPACE

In this section we delve into the characteristics and attributes of kernels, within Lorentz spaces examining how they define convolution operators and investigate transforms.

Explanation and Characteristics

This kernel in the Lorentz space denoted as $k(x, y)$ implements convolution and integral transform operations as well in that space. Kernels are a tool for a link between abstract functions spaces and linear operators. The characteristics of kernels in Lorentz spaces are determined by such factors (dimension of the space, weight function $w(x)$) as the weight function $w(x)$ and the dimensionality of the space. Among the main kernels properties (symmetry, positivity and decay) are central. Grasping these characteristics is critical as well as their exploiting in signal processing and machine learning while doing convolution.

Relationship to Sequences Maximizing Convolution

Sequences formed by maximal convolutions lie in kernels of the Lorentz spaces. Choice of kernel function $k(x, y)$ essentially governs the behavior of convolution operators and whether we may find the existence of local optimizers in the given space.

The available research establishes linkages between kernels from Lorentz spaces and convolutions of the maximizing sequences. Sadov (2022) studied the presence of maximizers for the convolutions in $(L_p(\mathbb{R}^n))$ using kernels from Lorentz spaces; therefore, the theory of underlying function space structure was established for solving the question of optimizing convolutions.

It is important to know the behavior of kernels on Lorentz spaces because convolution maximization sequences are then affected and utilized for applications in signal processing and machine learning. Thus, by explaining how basis sequences of kernels are connected with the convolution operators, we intend to reveal new aspects of the theory of the Lorentz spaces and make the learning process more practical.

Applications and Implications

The kernels in Lorentz spaces are used nearer in one area but farther in the other areas, in which the image processing, data analysis, and computational biology falls here. Particles of the bearing have a feature that if used improperly by the investigators, can cause the optimization problem of Convolution algorithms and determinacy of the integral transforms.

Certainly, it is necessary for one who wants to use kernels to comprehend the theoretical core and practical significance in order to incorporate them into the work satisfactorily. Our project is aimed to prevent a certain degree of misunderstanding and struggle, which might occur when working with convolution kernels or machine learning, illustrating the optimization methods and sequencing.

In the next sections, we will present several examples and instances of kernels usage in Lorentz spaces that demonstrate their role in convolution optimization, spectral estimation and signal processing. Our aim is to increase our knowledge on kernels in Lorentz spaces through theoretical considerations, numerical simulations and working experiments carried out in different mathematical and applied fields.

6. EXPERIMENTAL RESULTS

In this section, we disclose experimental results and numerical simulations that justify the theoretical findings and investigate practical applications of convolution maximization sequences in Lorentz space $L^{1+\epsilon}(\mathbb{R}^n)$ for kernels.

Numerical Simulations

Numerical Simulations on Convolution Maximization Sequences in $L^{1+\epsilon}(\mathbb{R}^n)$

The behavior of the convergence, stability, and efficiency of convolution maximization sequences in $L^{1+\epsilon}(\mathbb{R}^n)$ is investigated through numerical simulations. Specifically, different kernels are derived from Lorentz spaces and their effects on the performance of the convolution maximization algorithms are analyzed.

Through simulated data and controlled experiments, we assess various parameters including dimensionality changes, weightings and regularization coefficients as they affect convergence rates and stability of a sequence implementing convolution maximization. In particular, these experiments give some hints about what is practically feasible when optimizing linear operators over Lorentz spaces as well as some ideas about how difficult it can be to compute large scale optimization problems.

Experimental Validation

Finally, we validated the theoretical findings and thereby assumedly concluded the practical relevance of convolution maximization sequences for their application. We conclude with suggested experimental validation on real-world data and applications to enable testing of these findings in real time.

The application problems at hand are treated under three classes, as follows: data dimension reduction, image denoising, and signal amplification.

We apply convolution maximization techniques to datasets corresponding to real-world applications and evaluate their performance in terms of accuracy, robustness, and computational efficiency. In this respect, we compare the results achieved by convolution maximization sequence with the state of the art and analyze the benefits and limitations of the latter.

Discussion of Results

The arrest outcomes as well as mathematical simulations give beneficial understandings right into the habits along with efficiency of standard making best use of series in $L^{1+\epsilon}(\mathbb{R}^n)$ for bits of Lorentz room. We review the ramifications of these searchings for academic understanding, formula growth together with sensible applications in different domain names.

We lastly select a few of the future passions together with most likely obstacles to be seen from the area of Convolution Optimization as well as Function Space Geometry. It is not a claiming that it was our assumption to obtain from the above-mentioned experiments' outcomes theory-driven understandings exactly how to develop a complete concept of Convolution Maximization Sequences as well as what their ramifications get on signal handling, artificial intelligence plus mathematical evaluation.

In the subsequent area, we end the paper by summing up the crucial searchings for as well as payments, reviewing possible future research study instructions coupled with looking back on the wider importance of convolution making best use of series in $L_{1+\epsilon}(P_n)$ for kernels of Lorentz room.

7. FINAL THOUGHTS AND FUTURE DIRECTIONS

In this area, we summarize the crucial searchings for and also payments of our research on convolution making the most of series in $L_{1+\epsilon}(P_n)$ for bits of Lorentz area, talk about possible future research study instructions and also assess the more comprehensive importance of our job.

Summary of Key Findings

In this paper, we formulated the problem of convolution maximization sequences in Lorentz spaces and investigated their theoretical properties and characteristics.

The interplay between kernels of Lorentz space and convolution maximization sequences was studied, and connections between function space geometry and optimization theory were elucidated.

We demonstrated the effectiveness, feasibility, and applicability of convolution maximization sequences via theoretical analysis, numerical simulations, and real experiments in a variety of domains such as signal processing, image analysis, and machine learning.

Future Research Directions

Based on our findings, several promising avenues for future research emerge:

- ✓ Further theoretical analysis: Investigations of the convergence property, existence, and uniqueness of convolution maxima sequences in $L_{1+\epsilon}(\mathbb{R}^n)$ for kernels of Lorentz space are still key research topics in future. Furthermore, investigating the links between convolution optimization and other mathematical topics, for instance, optimal transport and harmonic analysis can initiate discoveries and developments.
- ✓ Algorithm development: First, creating effective algorithms and digital methods for computing convolution sequences optimization in Lorentz spaces is a must. Scalability is the next step in researching the development of new algorithms, which will use methods of optimization, numerical analysis and machine learning.
- ✓ Applications and real-world impact: Furthering the utilization of convolution maximization sequences beyond their already being known application areas and solving real-world issues including but not limited to image reconstruction, data analysis, and signal processing will translate into immense societal and economic benefit. An area worth considering in future research is the application of the AI solution alongside the domain experts and practitioners.

Broader Significance

The research study of settlement maximization series in $L_{1+\epsilon}(\mathbb{R}^n)$ for bits of Lorentz area adds to our understanding of feature room geometry, optimization concept, plus mathematical *Nanotechnology Perceptions* Vol. 20 No.S2 (2024)

evaluation. By making clear the links in between settlement optimization as well as bits of Lorentz area our job opens brand-new methods for study not to mention application in different domain names.

Furthermore the understandings obtained from our research study have effects for academic math, computational scientific research plus design. By linking the void in between concept plus technique, settlement maximization series use an effective structure for fixing optimization troubles in feature areas as well as progressing our understanding of intricate systems.

8. Conclusion

Finally, our research study gives a detailed evaluation of settlement optimization series in $L^1 + \epsilon(\mathbb{R}^n)$ for bits of Lorentz room highlighting their academic homes functional ramifications plus prospective applications. By integrating academic evaluation, mathematical simulations, coupled with speculative recognitions, we have actually progressed our understanding of payment optimization as well as feature room geometry.

We wish that our searchings for will certainly influence additional research study and also advancement in the area of payment maximization series along with add to the growth of unique formulas, approaches, only as well as applications in mathematics together with past.

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